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USING HEURISTIC STRATEGIES TO PROMOTE MATHEMATICS PROBLEM SOLVING ABILITY OF GRADE 10 STUDENTS

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Abstract

The main purposes of the research were to investigate how to teach Heuristic strategies to promote students solving the problem, and to study the ability of tenth grade students who had been taught to solve mathematic problems by utilizing the Heuristic method. The target group consisted of 47 grade 10 students at Wattanothai Payap school, Chiang Mai Province, Thailand. The researcher conducted classroom action research with 3 cycles. Instruments included lesson plans, students' written work, and the problem-solving ability test. Data was analyzed by frequency, percentage, and content analysis.

The results showed that 1) The teaching approach utilizing Heuristic strategies used in this research consisted of the following steps: the teacher encouraged the students to experiment by using their own strategies, then challenged them to utilize the complicated strategies, Guess-check-revise, Systematic experimentation (making a table, equation), and Use of graph of function. 2) Results from the test showed that 35 students of the 47 students (74.46%) passed the criteria (70% of total score). The strategies most students used were Systematic experimentation, and

Guess-check-revise, and Use of graphs of function, respectively. From the results, the researcher learned that the key success factors that made students dare to solve problems, were to give them more tools to use and more opportunities to learn from each other. Students were able to freely choose their own way to solve problems without any fear.

Keywords

Heuristic Method, Problem solving, Functions and Relations

1. Introduction

Problem – solving process is one of the most important aspect of mathematics that teachers should be concerned with (Schoenfeld, 1985). It is a widespread view that problem solving skills help individuals easily overcome problems they encounter in their daily lives. Numerous studies in mathematics education, Pape and Wang (2003) and Ridgway et al. (2002) hold strategy use as being central to solving mathematical problems and achieving success in solving problems is directly related to the choice of the appropriate strategy (Cai, 2003). One of the general strategies under element is Heuristic (Kiong et al., 2014).

The Heuristics teaching method is one of the most important teaching methods that can be utilized in teaching and learning problem-solving in Mathematics (Apostol, 2017). According to Shoenfeld (1985) "if we really expect students to use a heuristic strategy, we must teach it". According to Carson (2007) teacher should teach Heuristic to students and give the students the opportunity to freely solve the problem, then they will know how significance of each strategy. Teachers should illustrate the importance of heuristics to solve math problems. By doing so, students will assimilate these heuristics as their stock of tools which can be readily called into play at any time (Kiong et al., 2014). Polya (1954) suggested strategies such as: guess and check, look for patterns and make suppositions; work backward; restate the problem, simplify the problem, and solve part of the problem, etc. Fan and Zhu (2007) explained specific heuristic strategies such as Act it out, Change your point of view, Draw a diagram, Guess and check, Logical reasoning, Look for a pattern, Make a systematic list, Make a table, Make suppositions, Restate the problem, Simplify the problem, Solve part of the problem, Think of a related problem, Use a model, Use an equation, Use before-after concept and Work backwards. On the one hand, Kiong et al. (2014) emphasized on Trial and error, Draw a diagram, Make a table, Look for patterns, Test and simulate, Try a simpler problem, Analogy, and Work backwards. Novotna (2014) defined heuristic strategies such as Strategy of analogy, Guess-check-revise, Systematic experimentation, Problem reformulation, Solution drawing, Working backwards and Use of graphs of function. In addition,

Krulik and Rudnick (1996) cited in Apostal (2017) suggested Computing or Simplifying, Using a formula, Making a model or diagram, Making a table, chart, or list, Guessing, checking, and revising, Considering a simple case, Eliminating, and Looking for patterns.

The researcher was a preservice teacher teaching tenth grade students at Wattanothai Payap school, Chiang Mai, Thailand. The problems were when the students faced with a difficult problem, some of them could solve, while others would just leave the problem without even attempting to solve it. On the other hand, it was found that students who could solve the problem utilized only one strategy. For analyzing the cause, the researcher found that students lacked tools to solve the problem, and they were hindered by only having strategy to solve problems. So, the researcher sought to convey them that there are many ways to solve problems, using Heuristic strategies. From many studies, it was found that Heuristics was an essential part of problem solving that all students should learn. The researcher adapted the methods from Novotna (2014) such as Guess-check-revise, Systematic experimentation, Problem reformulation, Solution drawing, Working backwards, and Use of graphs of function. We expected that if students know many ways to solve, they might be able to choose the good way for them and solve the problem successfully. Another important things was how the teacher approach these strategies to the students to make them more understandable and select the appropriate way.

Hence, this research aimed to investigate how to teach Heuristic strategies to promote the students' abilities to solve problems, and to study the results of the tenth grade students who had been taught to solve Mathematic problems by utilizing the Heuristic method.

Thus, the research questions were:

1) How to teach the heuristics strategy to promote students solving problems?
2) How do students use the heuristics strategy to solve problems?

2. Theoretical Framework

2.1 Heuristic Strategies

Teaching of problem-solving heuristics even in the elementary and high school years should be encouraged in mathematics classrooms (Apostol, 2017). Many researchers explained and defined heuristic strategies used for their solution. Polya (1954) suggested strategies for solving the problems 1) To make a calculated guess are Guess and check, Look for pattern, and make suppositions 2) To go through the process are Act it out, and Work backwards. 3) To change the problem are Restate the problem, Simplify the problem, and Solve part of the problem.

Fan and Zhu (2007) lists of heuristic strategies such as Act it out, Change your point of view, Draw a diagram, Guess and check, Logical reasoning, Look for a pattern, Make a systematic list, Make a table, Make suppositions, Restate the problem, Simplify the problem, Solve part of the problem, Think of a related problem, Use a model, Use an equation, Use before-after concept and Work backwards.

Kiong et al. (2014) studied how heuristics enhance the success of Mathematics problem solving. They founded the most common heuristics used un Mathematics include 1) Trail and error: The problem solver solves the problems start with the trail and checks the plausibility of the answer obtained. 2) Draw a diagram 3) Make a table: All possible cases for a problem will be listed in a table. 4) Look for patterns: This strategy can guide a problem solver to reach the answer by existing some patterns in the information given. 5) Test and simulate 6) Try a simpler problem: The difficult problem can be broken up into a number of simpler problems. The solver will solve in the easy part first then they will use this result to obtain a solution of original problem.
7) Analogy 8) Work backwards: The problem solver have to clarify the given information and the goal of a problem then start to solve the problem from the goal.

This research follows heuristics strategies of Novotna (2014) using the following heuristic strategies: "strategy of analogy", "guess–check–revise", "systematic experimentation", "problem reformulation", "solution drawing", "working backwards", and "use of graphs of functions". They were explained as follows:

Strategy of analogy: By finding problems that resemble each other, when we solve one of the problems, we can apply the same strategy to solving the other problems.

Guess – check – revise: We use our past experiences to help us guess the result, followed by checking to see if it is correct. Then future guesses are made taking into account the previous guess. This continues until the correct result is found.

Systematic experimentation: This method utilizes experimenting to find the solution. We methodically change the numbers used in the algorithms until we find the correct answer.

Problem reformulation: This strategy involves taking an existing problem and creating a new problem that is easier to solve. By solving the new problem, it can assist in solving the original problem.

Solution drawing: With solution drawing, we draw a visual depiction of the problem they are trying to solve. By depicting the problem, and sometimes the end result, it can help with understanding the problem. This has been especially effective in tackling new concepts.

Working backwards: This commonly used strategy, starts with the final solution, and then using inverse operations, works backward step by step to get back to the beginning. This is especially helpful when there are multiple steps.

Use of graphs of functions: When a problem includes functions, it has been found that graphing can substantially assist in problem-solving.

In this research, the researcher used 4 strategies from the framework to taught content of Functions such as Guess-check-revise, Systematic experimentation, Solution drawing, and Use of graphs of functions.

2.2 Action Research

Research proceeds as a cycle of joint planning, action, observation and reflection, where the reflection phase paves the way for further cycles of planning, acting, observing and reflecting in a spiral of learning. The final stage is a reflection, about what has been achieved and identifying possible ways of moving forward. Depending on the outcome, another cycle of planning, acting, observing and reflecting may be set in motion (Kemmis and McTaggart, 1988).

Shown as a Figure 1.

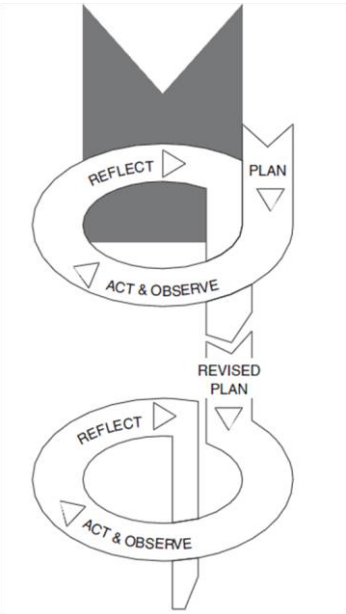


Figure 1: *Action Research Cycle (on the basis of Kemmis & McTaggart, 1988)*

3. Methodology

In this research, the researcher conducted 3 cycles of action research with planning, action, observation and, reflection (Kemmis & Taggart, 1988).

3.1 Target Group of the Study

The target group was 47 grade 10 students at Wattanothai Payap school, Chiang Mai Province, Thailand.

3.2 Research Instruments

- **Lesson plans**

  The lesson plans of Functions and relations, Linear function, Quadratic function, Exponential function were included in 14 lesson plans during all 3 cycles. Lesson plans also included the problems, to demonstrate that problems can be solved in many ways to obtain the solution.

- **Students' written work**

  Students' written-work was used for reflecting how the students used strategies to solve the problems during each phase.

- **Problem solving ability test**

  The problem solving ability test was designed to include 3 problems to indicate how the students use strategies to solve the problems.

3.3 Data Collection

The students were taught through 14 lesson plans how to utilize Heuristic strategies, and then the researcher observed how the students implemented the Heuristic strategies. After the study was finished, the students were given the problem-solving ability test.

3.4 Data Analysis

Data were analyzed both quantitatively and qualitatively. The researcher used frequency, percentage, and content analysis, in analyzing the data.

4. Results and Discussion

4.1 Teaching Approach in the Classroom

The teaching approach is show in Figure 2.

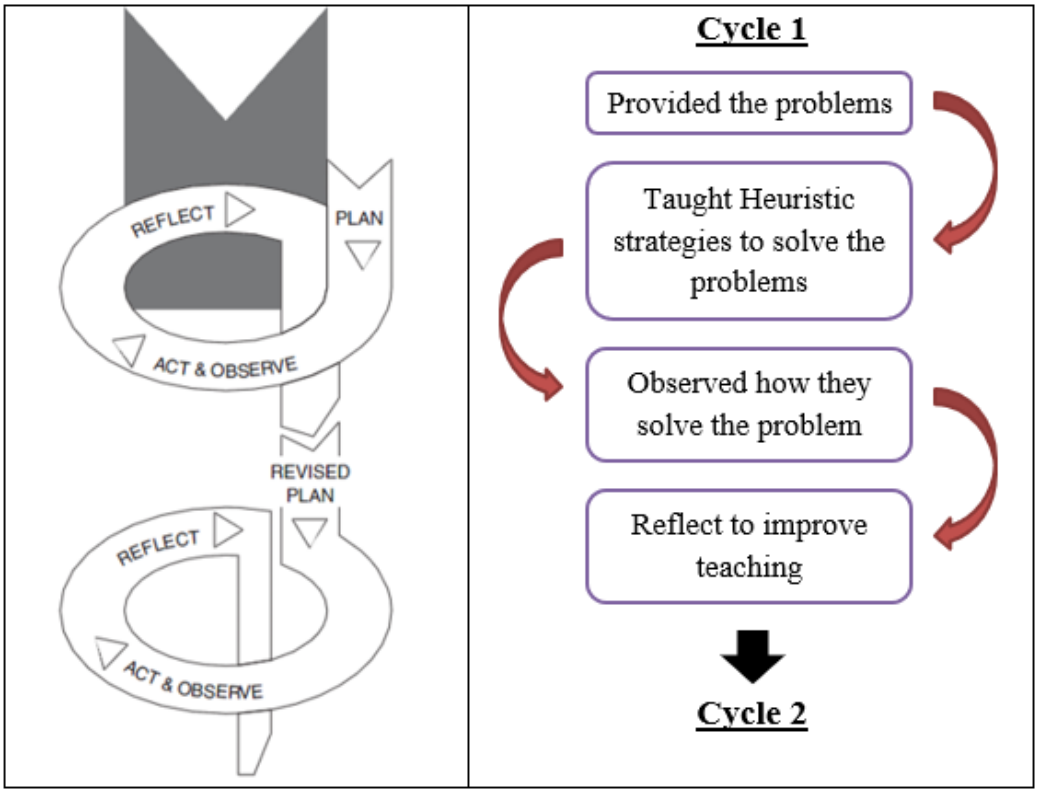


Figure 2: *Teaching Approach in the Classroom*

To investigate the strategies that students might have known, initially, the teacher (as a researcher) provided problems about Functions for students to solve using their own strategies. The results showed that most students used Guess-check-revise, but some were not able to reach the solution. Some students used making equation, some were able to solve the problems, while others were not.

In cycle 1: Linear function. Students were faced with the problems, which they then tried to solve by themselves. Then the researcher tried to introduce the strategy students used and explain them the way to reach the solution and followed by Systematic experimentation and Use of graphs of functions.

In cycle 2: Quadratic function. After the researcher provided the problem on quadratic function and then observed, the students still used Guess-check-revise. The researcher revealed the limitations of the Guess-check-revise strategy in order to challenge them to seek for other strategies. So sequence of teaching the strategies were started with the Systematic experimentation, Use of graphs of functions, followed by solution drawing.

Cycle 3: Exponential function. In the beginning, the students used Systematic experimentation, by making a table, the researcher then introduced them to alternative approaches to reaching the solution by utilizing Systematic experimentation-using equations and Use of graphs of functions.

4.2 Student's Heuristics Strategies

4.2.1 During Learning in the Classroom

The trend of using Heuristic strategies is shown in Table 1.

Chart 1: *Frequency of Heuristic Strategies used by Students in Class for in Solving Problems*

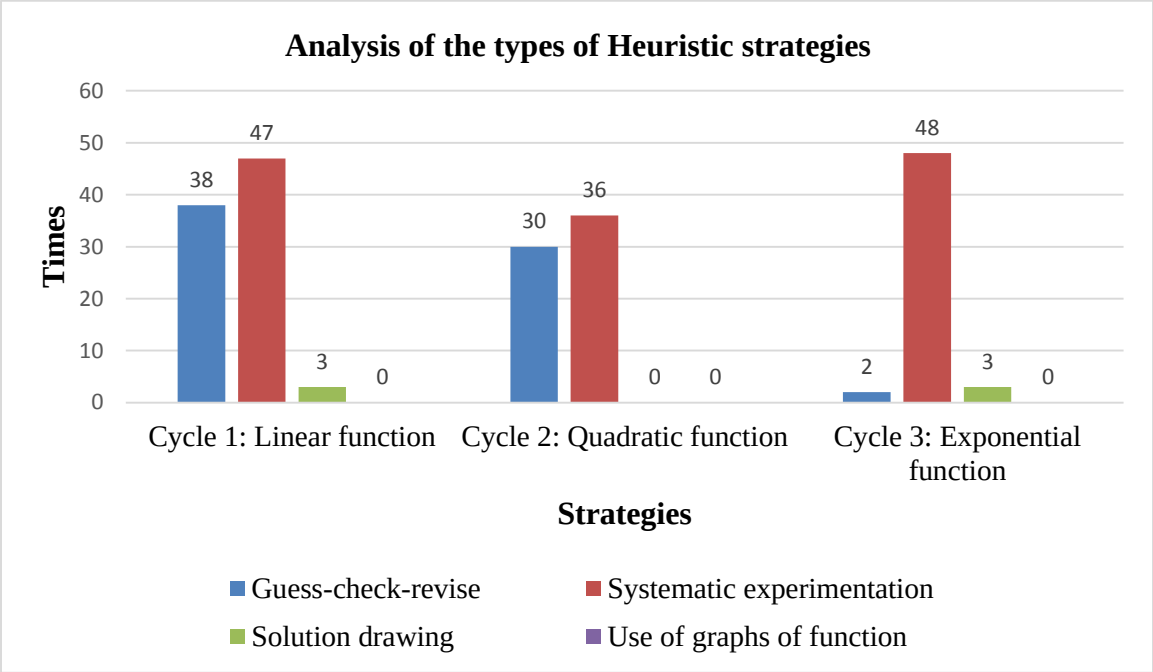


Chart 1 shows that during learning in the classroom, for cycle's 1 and 2, students used Systematic experimentation to solve the problems most frequently. The second strategy was Guess – check – revise.

In cycle 3, the Exponential function, students still used Systematic experimentation the most, with Solution drawing coming in second.

4.2.2 From the Test

Chart 2: *Frequency of Heuristic Strategies used by Students in problem Solving Ability Test*

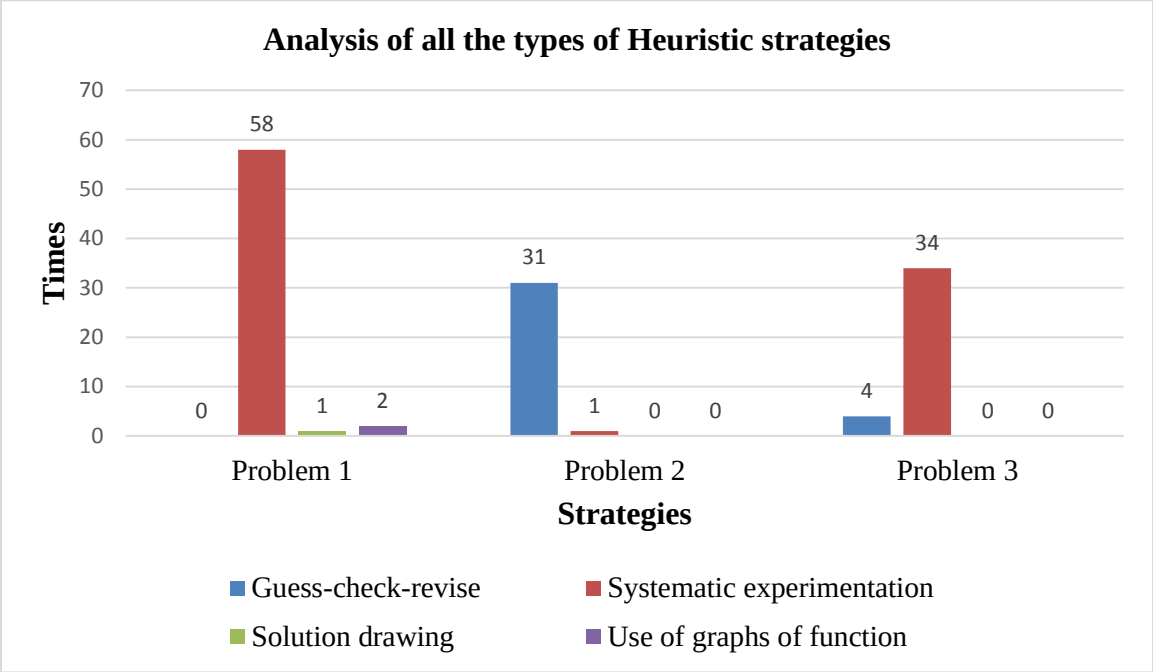


Chart 2 shows the frequency of students using strategies to solve problem on the test. For Problem 1 and Problem 3, students used Systematic experimentation to solve the problems most frequently. For the Problem 2, the students used Guess-check-revise to solve the problem most frequently and Systematic experimentation used less frequently.

The results of the problem-solving ability test showed that the average score for the test was 12.18 out of 15 score. Results exceeded the teacher's expectations of the students getting at least 70 percent of the total score. The students surpassed the expectations by achieving of 74.46 percent with thirty-five of 47 students achieving a score of more than 70 percent with 17 achieving a perfect score of 15. The students' test scores showed whether students were capable of choosing the perfect strategy to solve the problem or not. All of them were able to face the problems and try to solve them. The example of students' responses was as follows:

Problem 1: Supposed that growth rate of Pine tree is $y = 2x + 5$ and Eucalyptus tree is $y = 3x + 2$, where x represents a period (month) and y represent a height (centimeter). (1) Find when the height of the Pine and Eucalyptus will be the same. (2) What will their height be, five mounths after they were planted?

- Systematic Experimentation

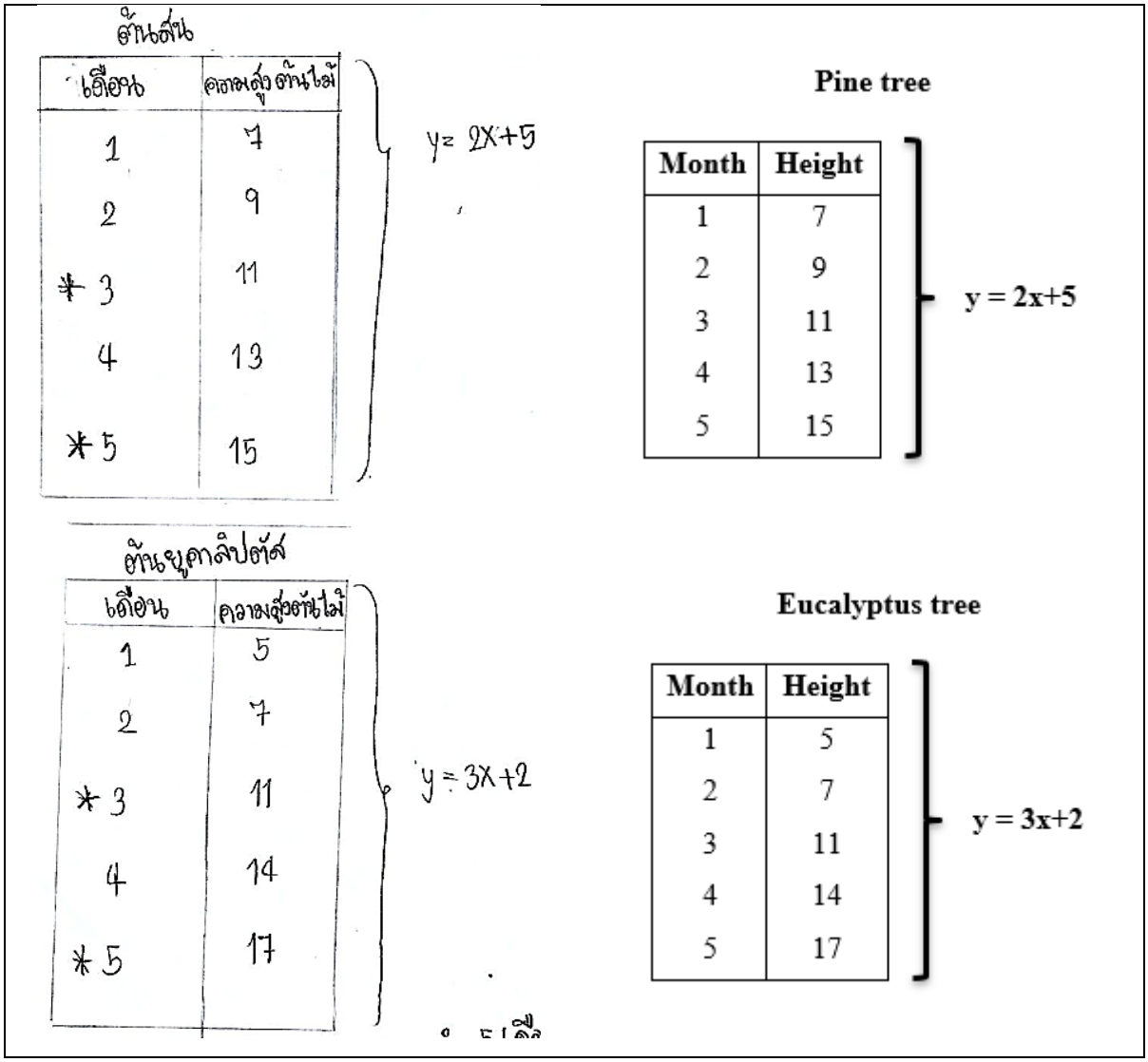


Figure 5: An *Example of a Student's solution using Systematic Experimentation*

Figure 5 shows how students solved the problem using Systematic experimentation. Making a table is one strategy that can be used to solve the problem using Systematic experimentation. We could see the sequence of the method the students used to solve the problem, and we found that it was clearer than Guess – Check – Revise.

- Solution Drawing

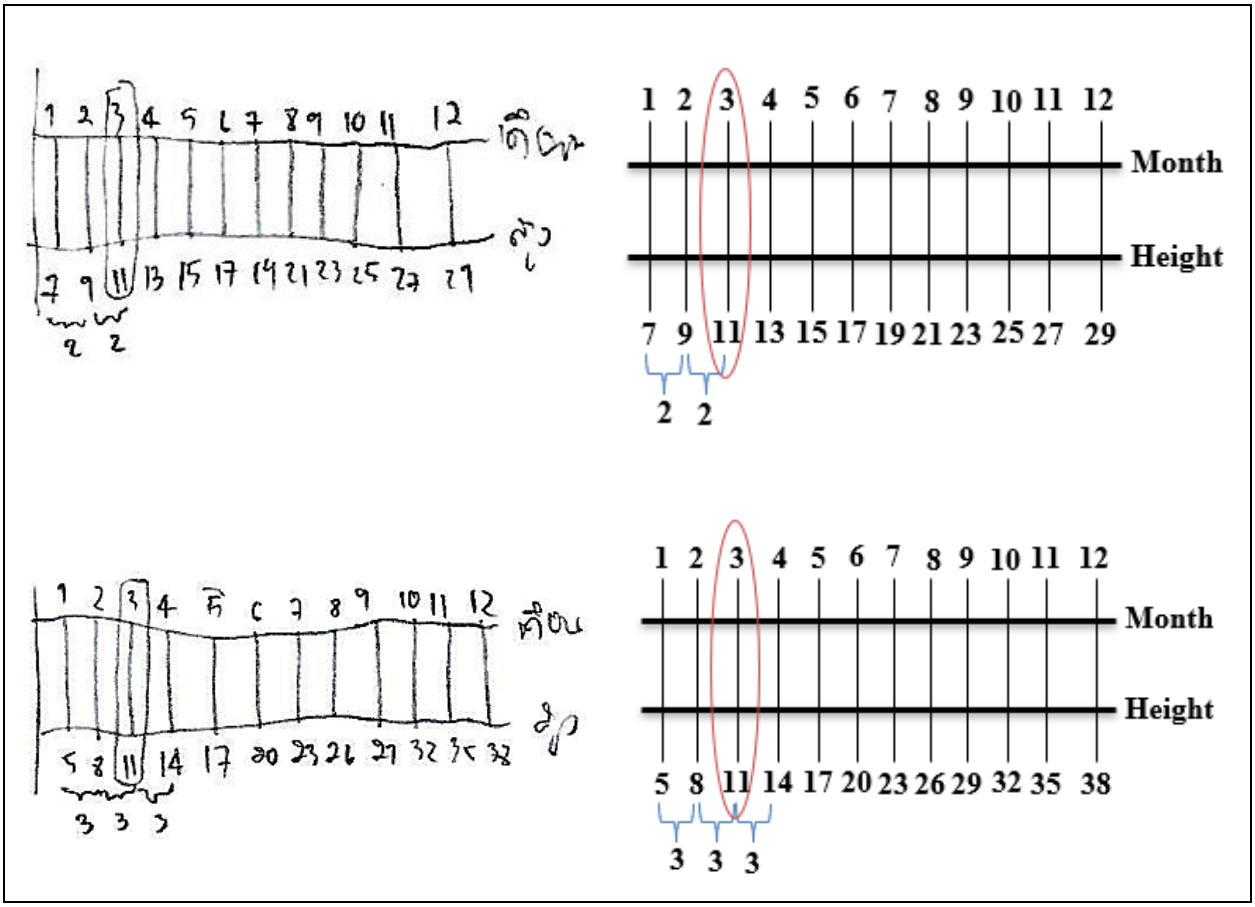
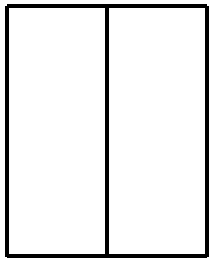


Figure 6: *An example of a student's solution using Solution drawing*

Figure 6 shows how students solved the problem with Solution drawing. Students solved the problem by drawing something that shows how to achieve the solution of the problem.

Problem 2: Two rectangular corrals are to be made from 100 meters of fencing as seen below.



If the rancher wants the total area to be maximized, what dimensions should be used to make the corral's?

- Guess – Check – Revise

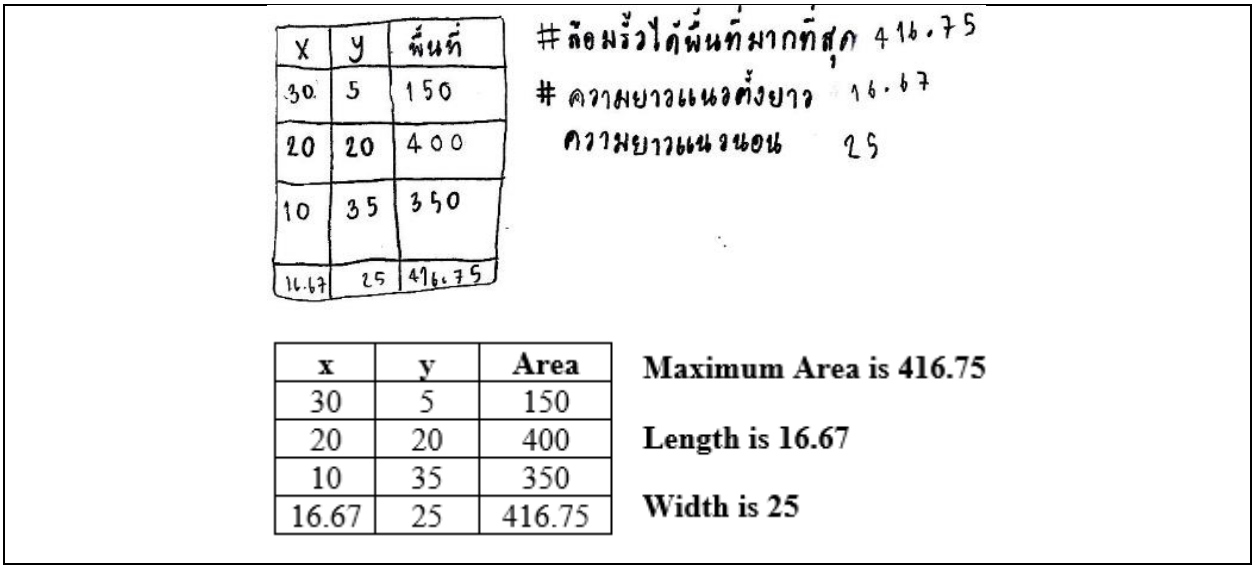


Figure 7: *An Example of a Student's Solution by using Guess – Check – Revise*

Figure 7 shows how students solved the problem using Guess – Check – Revise. Students' used a random number to guess the answer to the problem (Student guessed the y variable), and then they checked the number that they guessed. If the number appears to work in the problem, maybe it is the answer to that problem. So, they just answered the problem with that number.

- Systematic Experimentation

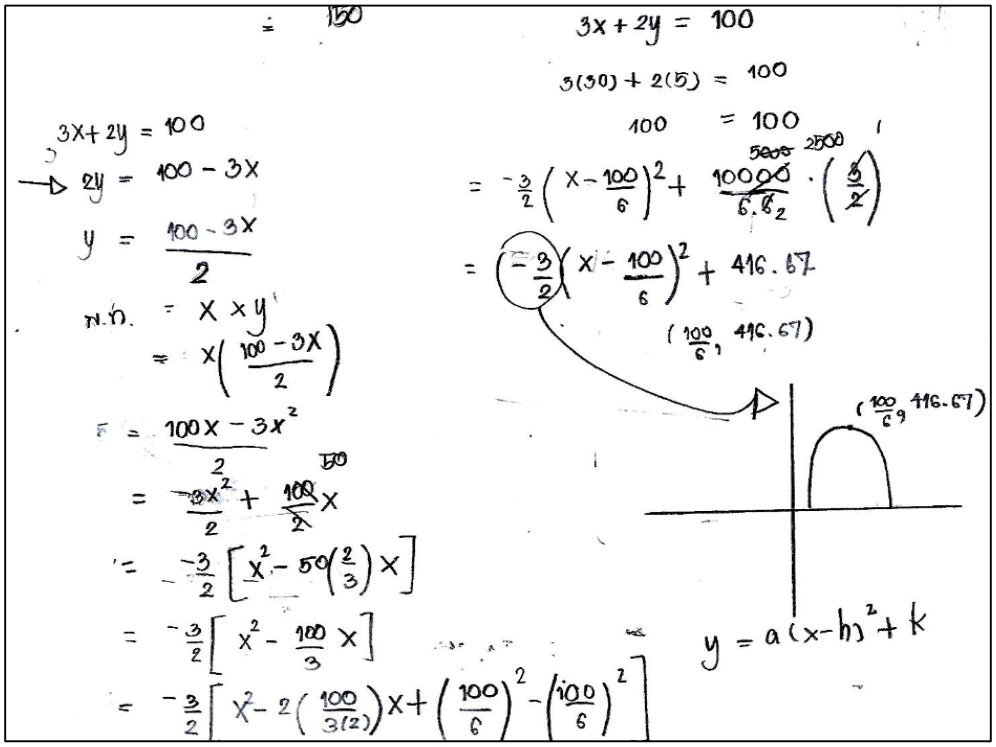
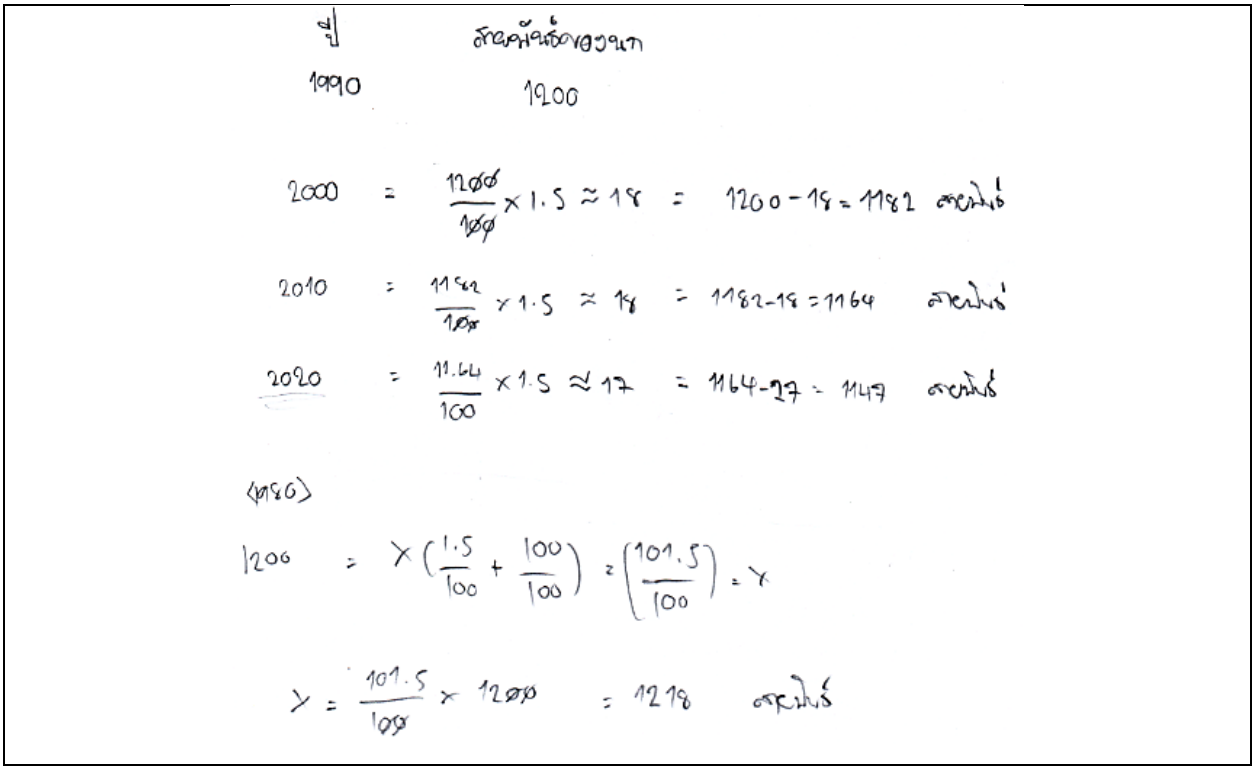


Figure 8: *An Example of a Student's Solution using Systematic Experimentation*

Figure 8 shows how students solved the problem using Systematic experimentation. Students solved the problem using algebra, which is a more formal equation solving process in mathematics. We were able to see, through the process, how the students comprehended the problem, also.

Problem 3: In 1990 a rural area has 1200 bird species. If the species of birds are becoming extinct at the rate of 1.5% per decade (ten years), how many bird species will be left in the year 2020? And at the same rate, how many were there in 1980?

- Systematic Experimentation



Year	Bird species
1990	1200
2000	$=\frac{1200}{100}\times 1.5\approx 18$ then $1200-18=1182$ species
2010	$=\frac{1182}{100}\times 1.5\approx 18$ then $1182-18=1164$ species
2020	$=\frac{1164}{100}\times 1.5\approx 17$ then $1164-17=1147$ species
(1980)	

$$1200 = x\left(\frac{1.5}{100}+\frac{100}{100}\right)=\left(\frac{101.5}{100}\right)x$$

$$x=\frac{101.5}{100}\times 1200=1218 \text{ species}$$

Figure 9: *An Example of a Student's Solution using Systematic Experimentation*

Figure 9 shows how students solved the problem using Systematic experimentation. Students solved the problem by making a table, which is a more formal equation solving process in mathematics. We were able to see how the students understood the problem also.

Chart's 1 and chart 2 show, that during both, learning in the classroom and the test, the strategies most frequently used were Systematic experimentation followed by Guess-check-revise.

5. Conclusion

In conclusion, the teaching approach by utilizing Heuristic strategies used in this research employed the following steps: the teacher encouraged students to experiment by using their own strategies, then challenged them to utilize the complicated strategies, Guess-check-revise, Systematic experimentation (making a table, equation), and Use of graph of function. Results from the test showed that 35 students from 47 students (74.46%) passed the criteria (70% of total score). The strategies most students used were Systematic experimentation, Guess-check-revise, and Use of graphs of function, respectively.

From the results, the researcher learned that the key success factors that make students dare to solve problem were to give them more tools to use and give more opportunities to learn from each other. Students were able to freely to choose their own way to solve the problem without any fear. As Villonez (2018) said, students learn from sharing with each other via observation, imitation, and modeling, that might help them to construct their own way. Therefore, the teachers

should create more opportunities for students to participate in the teaching-learning environment and share their own strategies (Abeygunawardena & Vithanapathirana, 2019).

As far as teaching strategies for the students, the researcher found that we should start teaching by giving them opportunities to create their own strategies. Then gradually they would approach the easiest way, using Guess-check- revise, to make them more confident to solve the problem. Then the teacher tried to make them see the limitations of the old strategies and show them a new way, so they could understand why they might not use the same way with the other problems. There could be a much more appropriate way.

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