Flordeliza P. Ferrer, 2016

Volume 2 Issue 1, pp. 310-324

Year of Publication: 2016

DOI- http://dx.doi.org/10.20319/pijss.2016.s21.310324

This paper can be cited as: Ferrer, F. P., (2016). Investigating Students' Learning Difficulties in Integral Calculus. PEOPLE: International Journal of Social Sciences, 2(1), 310-324.

INVESTIGATING STUDENTS' LEARNING DIFFICULTIES IN INTEGRAL CALCULUS

Flordeliza P. Ferrer
Pamantasan ng Lungsod ng Maynila (University of the City of Manila), Philippines
fpf92@yahoo.com.ph

Abstract

The study aims to investigate the students' learning difficulties in Integral Calculus, specifically in calculating integrals. The mixed methods research design was employed to gather quantitative data from the students' answers in the examination and collect qualitative data from them through open-ended interviews and scrutiny of their solutions. Findings of related previous investigations were documented to understand more the nature of the difficulties in the subject as experienced by others. Participants of the study were given the examination in indefinite integrals composed of items that require solely the use of integration formulas and items in which integration techniques should be applied. The quantitative data disclosed, after subjecting the index of difficulties to statistical treatment, that the learners have experienced the same level of difficulties in dealing with the two types of integrals. The qualitative data revealed, as viewed by the participants and as reflected in their solutions, that many of the errors pertain more to the learners' ability in trigonometric manipulation rather than with the integration per se. It was further concluded as highlighted from previous studies, that the learners' difficulties in *integration are evidently attributed to the inherent mathematical knowledge and skills acquired by the students from basic mathematics.*

Keywords

Indefinite Integral, Integral Calculus, Integration Formula, Integration Technique

1. Introduction

Advancements supported by the methods of calculus have been made and continued in various fields such as education, business, engineering, science and technology. Because calculus is used in a broad range of disciplines for a variety of purposes, it is considered to be an important subject of study for both high school and college students (Moskal, 2002). Calculus is a branch of mathematics concerned with the calculation of instantaneous rates of change, known as Differential Calculus, and the summation of infinitely many small factors to determine some whole, known as Integral Calculus (Berggren, 2016).

Integral Calculus is generally concerned with the determination, properties and applications of the integrals of functions. It is used in the calculation of area bounded by curves, volume of a solid of revolution, centroid, moment of inertia, fluid pressure, work, etc. (Peterson, 1968; Leithold, 1996; Stewart, 2010). Students must have passed the pre-requisite subjects Algebra, Trigonometry, Analytic Geometry and Differential Calculus for them to qualify to take this subject. This course is a gateway to more advanced courses in mathematics such as Differential Equations, Vector Analysis and Complex Analysis.

An increasing number of investigations have shown that students have difficulties in understanding the concept of Integral Calculus (Orton, 1983; Tall, 1993; Kiat, 2005; Metaxas, 2007; Yee & Lam, 2008; Mahir, 2009; Souto & Gomez-Chacon, 2011; Usman, 2012; Salazar, 2014; Zakaria & Salleh, 2015). The analysis of calculus performance of the students at the university, particularly in Integral Calculus topic was found to be low (Zakaria & Salleh, 2015). Many students cannot achieve a deeper understanding; they find calculus very hard and abstract (Zhang, 2003, as cited by Mokhtar et al., 2010 & Salazar, 2014). Unless a strong foundation in its pre-requisite skills is achieved, the students' performance in Integral Calculus will still be low (Salazar, 2014). Considering the importance of this subject in academic undertaking, the researcher was prompted to investigate the students' learning difficulties in Integral Calculus, specifically in calculating integrals which integration requires the use of formulas and techniques. This study will provide essential feedback for the learners and educators recognize and understand the nature of difficulties in learning and teaching indefinite integrals.

2. Methodology

The study employed the convergent mixed methods approach of research design, where the investigator, collected both quantitative and qualitative data, analyzed them separately, and then compared the results to see if the findings confirm or disconfirm each other (Creswell, 2014). The quantitative data were gathered by administering an examination on indefinite integrals to participants and then qualitative data were considered in the next phase by conducting open-ended interviews and inspection of the participants' solutions in the test papers.

One hundred (100) engineering students took the examination on indefinite integrals. The nature of integrals given can be described as more of a general form of integration, and can be interpreted as the anti-derivative of the given function. The examination is composed of forty (40) items, randomly and equally distributed to integrals in which integration techniques are permissible and not. No identification was given as to the type of each integral, neither the information on the integration formula to be used, nor the integration technique to be applied.

There are twenty (20) integrals which require no integration technique, but the use of the following formulas: integration of powers; integration of exponential functions; integration of logarithmic functions; integration of trigonometric functions; transformations of trigonometric integrals; and integrals giving inverse trigonometric functions. The remaining twenty (20) items require the application of the following techniques: integration by parts – to be used when the integrand is the product of an easy function and a hard one; algebraic substitution – to be applied when the integrand has algebraic expressions in radical form; trigonometric substitution – to be employed when the integrand contains expression of the form square root of the sum/difference of squares; integration of partial fractions – to be used to express algebraic rational expression as a sum of proper rational expressions; reciprocal substitution – to be applied to integrate algebraic functions; and half-angle substitution – to be employed to integrate expressions in trigonometric form (Peterson, 1968; Leithold, 1996; Stewart, 2010). The sets of questions included in the test are the items that require the basic understanding of the learners to perform integration in Calculus.

The index of difficulty per item in the examination was computed using the ratio of the number of students who got incorrect answer and the total number of participants in the study. The level of difficulty based on the computed index was classified as follows: very difficult (0.81-1.00); difficult (0.61-0.80); average (0.41-0.60); easy (0.21-0.40); and very easy (0.01-0.20). The mean difficulty index for each type of integrals were computed, compared and analyzed using the t-test for independent samples.

It may be noted that mixed methods research contain a theoretical framework within which both quantitative and qualitative data are collected. With this note, the researcher conducted open-ended interviews to get the detailed views of the participants and their test papers were scrutinized to examine further the common errors they committed in the integration. Moreover, the theoretical framework was drawn from the perspectives of other investigators who conducted similar study in order to get a deeper understanding of the nature of the difficulties of the learners in integration as experienced by others.

3. Results and Discussion

3.1 Students' Learning Difficulties in Integration Formulas

In all types of integrals, regardless whether integration techniques are permissible or not, the use of appropriate integration formula is a key to carry out integration successfully. The type of integral described in the following discussion is the one that does not require the use of the technique.

Table 1 shows the number and percent distribution of the items given in the examination classified according to the level and index of difficulties in the use of the following formulas: integration of powers; integration of exponential functions; integration of logarithmic functions; integration of trigonometric functions; transformations of trigonometric integrals; and integrals giving inverse trigonometric functions.

Table 1: *Distribution of items according to level and index of difficulties in the use of integration formulas*

Level of difficulties	Index of difficulties	Number of items	Percent of items
Very difficult	0.81-1.00	2	10 %
Difficult	0.61-0.80	7	35 %
Average	0.41-0.60	9	45 %
Easy	0.21-0.40	2	10 %
Very easy	0.01-0.20	0	0 %
Total		20	100 %

Among the twenty (20) integrals of such type, two (2) of them were classified as very difficult, with a computed index of 0.81-1.00 and seven (7) as difficult, bearing an index of 0.61-0.80. This comprised 45% of the total items which require only the use of integration formulas (Table 1). To note, after examining the students' test papers, it appeared that the items with the difficulty index of above 0.60 are specifically on the use of the following formulas: integration of trigonometric functions; transformations of trigonometric integrals; and integrals giving inverse trigonometric functions.

The results of the interview with the students gave a confirmation that among the 20 items, they find difficulty in using the integration formulas mentioned in the preceding statement. When they were asked further to rank the six integration formulas according to their perceived level of difficulty when used in the integration, the response of the majority considered the transformations of trigonometric integrals as the most difficult; followed by integrals giving inverse trigonometric functions and integration of trigonometric functions (Table 2).

Table 2: *Difficult items identified in the use of integration formulas*

Difficult items identified in the use of integration formulas	Rank (1 as most difficult)
Items with transformations of trigonometric integrals	1
Items with integrals giving inverse trigonometric functions	2
Items with integration of trigonometric functions	3

Moreover, the participants of this study admitted that they have difficulty in recalling trigonometric identities, such as the transformation from product to sum of trigonometric functions, the half-angle identities, the double-angle identities, etc. It may be noted as claimed by the students that they have already learned those topics in their high school and college trigonometry. Moreover, learners also find hard to recall and apply the formulas of differentiation and integration of transcendental functions. Some of them expressed that they are confused of the formulas and oftentimes they interchanged the differentiation and integration formulas when transcendental functions are given.

The results of the previous studies were reviewed by the researcher to have a deeper understanding of the nature of the difficulties in the topic as experienced by others. It was established from those studies that many learning and achievement difficulties are directly related to inherent mathematical difficulties within specified concepts. For example, concerning calculus, the difficulties found in the research literature seem related to its fundamental notions such as function, derivative and integral (Lithner, 2011). Restricted mental images of functions are not always seen as provoking a difficulty in calculus, particularly when the subject is seen as focusing on the differentiation and integration of standard functions given as formulas (Tall, 1993).

Students seemed to have memorized a large range of formulas and procedures to be used for special types of integrands. The result was that they either could not recall the exact formula or were confused by the many formulas, thus, oftentimes they could not carry out the integration successfully (Yee & Lam, 2008). Learners have difficulty in recognizing equivalent equations and in making decision to which transformations are permissible and should be made in the context of the given equation (Maharaj, 2008). Varying the placement of the angle in trigonometry consistently affects student error and response patterns (Mikula & Heckler, 2013).

The learners are not familiar with the basic operational signs such as addition, subtraction, multiplication and division of trigonometric functions; hence they demonstrated a poor ability to simplify once they had completed differentiation (Siyepu, 2015). Most common among operational breakdown is inability to find the sum of two trigonometric fractions as well as clearing the resultant fraction. Moreover, learners find it hard to identify, recall, and use appropriate trigonometric identity (Usman, 2012). The largest numbers of errors committed in integration are technical errors which were primarily attributed to the students' lack of mathematical content knowledge in trigonometry. Accordingly, only a small number of students remembered the required formula (Kiat, 2005).

3.2 Students' Learning Difficulties in Integration Techniques

Although some integrals can be evaluated by direct application of the integration formulas, there are many for which this is not possible. In many instances, however, the integrals can be found by these formulas after suitable transformations have been made. The integrals of this type require the application of the following techniques: integration of powers; integration of exponential functions; integration of logarithmic functions; integration of trigonometric functions; transformations of trigonometric integrals; and integrals giving inverse trigonometric functions (Peterson, 1968; Leithold, 1996; Stewart, 2010).

Among the twenty (20) questions given in the test that require integration techniques, one (1) of the items was classified as very difficult, with an index of 0.81-1.00 and five (5) items as difficult, bearing an index of 0.61-0.80. This comprised 30% of the total items given in the examination where integration techniques are permissible (Table 3).

Table 3: *Distribution of items according to level and index of difficulties in the use of integration techniques*

Level of difficulties	**Index of difficulties**	**Number of items**	**Percent of items**
Very difficult	0.81-1.00	1	5 %
Difficult	0.61-0.80	5	25 %
Average	0.41-0.60	9	45 %
Easy	0.21-0.40	4	20 %
Very Easy	0.01-0.20	1	5 %
Total		20	100 %

To note, after examining the students' test papers, the items with difficulty index of above 0.60 are on the integrals that require the following integration techniques: integration by parts which they use when the integrand is the product of an easy function and a hard one; trigonometric substitution which they employ when the integrand contains expressions of the form square roots of sum/difference of squares; and half-angle substitution which they utilize to integrate expressions in trigonometric form.

The results of the interview with the participants gave an affirmation that among the six integration techniques, the learners really find difficulties with the techniques mentioned in the foregoing statement. When the students were further asked to rank the six techniques according to their perceived level of difficulty, the majority of them responded that using trigonometric substitution is the most difficult, followed by the use of half-angle substitution and integration by parts (Table 4). They further expressed that they know how to apply such techniques, but they have difficulty dealing with trigonometric relations when applied to right triangle. Students likewise admitted that they have taken already those topics in high school and college.

Table 4: *Difficult items identified in the use of integration techniques*

Difficult items identified in the use of integration techniques	**Rank (1 as most difficult)**
Items using trigonometric substitution	1
Items using half-angle substitution	2
Items using integration by parts	3

To understand more the nature of the difficulties of the students in the topic, the researcher highlighted the significant findings of other investigations that described similar cases in dealing with integration. The poor linkage between differentiation and integration, as concluded in previous study, implied that the learners could not check whether they had used appropriate methods or whether they had used the correct formula in integration (Yee & Lam, 2008). Students can apply with some facility the basic techniques of integration, but further probing showed that they have a fundamental misunderstanding about the underlying concepts of the application of the topic (Orton, 1983). Learners struggle with choosing the appropriate tool for the task out of the many equations related to potential. Once they have chosen an appropriate tool, some students struggle to set up or interpret the resulting line integral calculation (Pepper, Chasteen, Pollock, & Perkins, 2012).

The students could solve a number of problems by simply applying the rules that had been memorized and in some cases, incorrectly remembered (Huang, 2015). Oftentimes, they memorized the sequence of steps resulting to misconceptions due to misused data, logically invalid inference and technical mechanical error (Fui Fui & Shahrill, 2013). Many learners appeared to have little understanding of the underlying trigonometric principles and thus resort to memorizing and applying procedures and rules, while their procedural success masked underlying conceptual gaps or difficulties (De Villiers & Jugmohan, 2012).

3.3 Difference in Students' Learning Difficulties in Integration Formulas and Techniques

The index of difficulty computed for integrals that require solely the use of integration formulas (mean index of difficulty is 59.45, with a standard deviation of 15.01) and integrals that require the application of integration techniques (mean index of difficulty is 50.50, with a standard deviation of 18.99) were compared and analyzed. The t-value of 1.6536, with the corresponding p-value of 0.1069, disclosed that there is no significant difference between their mean difficulty indices (Table 5).

Table 5: *Comparison of index of difficulties in the use of integration formulas and techniques*

Classification of items	Mean difficulty index	Standard deviation	t-value	p-value	Conclusion
Use of integration formulas	59.45	15.01	1.6536	0.1069	Not significant
Use of integration techniques	50.50	18.99			

The students were asked further regarding their perceived common errors for which in many instances they commit in calculating indefinite integrals. The results of the interview showed that among the major steps needed to perform in order to integrate functions, oftentimes errors are in the manipulation of trigonometric functions; while the least error that they normally commit is in simplifying algebraic expressions. The response of the majority in the open-ended interview is summarized and presented in Table 6.

Table 6: *Students' perceived common errors in indefinite integrals*

Students' perceived common errors	**Rank (1 as most frequent)**
Manipulating trigonometric functions	1
Identifying the applicable integration formulas	2
Choosing the appropriate integration techniques	3
Determining the derivatives of the function	4
Simplifying algebraic expressions	5

Hirst concluded in her study that many of the structural errors observed in solving problems in calculus over many years are split into three categories: procedural extrapolation, pseudo linearity and equation balancing. Accordingly, this provides evidence that the types of error present in elementary mathematics continue into more advanced mathematics. Many of the errors pertain more to the students' ability in manipulation rather than with calculus per se. Many learners have been observed remarking on the difficulty of learning trigonometric functions (De Villiers & Jugmohan, 2012). Previous researches found gaps and omissions in a coherent pathway for students learning trigonometry between secondary school and college. These curricular discrepancies have the potential for being sources of difficulties for students in college programs (Byers, 2010).

Trigonometry served as an important precursor to calculus. Learning about trigonometric functions is initially fraught with difficulty. Trigonometric functions are typically among the first functions that students cannot evaluate directly by performing arithmetic operations (Weber, 2008). Trigonometry is an area of mathematics that students believe to be particularly difficult and abstract compared with the other subjects of mathematics. The most common errors that the students made are the improper use of equation, order of operations, and technical mechanical errors (Gur, 2009).

4. Conclusions

The students' learning difficulties in Integral Calculus are evidently based on the weak procedural knowledge of Trigonometry of the sample one hundred (100) students. The participants find difficulties whenever the given integrand is expressed in non-algebraic form, regardless of the required integration process. It was concluded based on statistical findings that learners experienced the same level of difficulties in calculating integrals applying either the integration formulas or the integration techniques. The computed indices revealed that the difficult items identified in the use of any of the two integration process both require the students' capability to operate and simplify non-algebraic functions.

It was found from the learners' solutions in the examination that many of the errors in the integration pertain more to their inability to transform a given trigonometric expression to its equivalent form that may subsequently permit integration. Supported by the results of the interview with the participants of this study, the students acknowledged their weaknesses in recalling trigonometric identities and in performing the basic fundamental operations involving non-algebraic expressions. The learners generally have the basic knowledge of the integration process, but short of the technical proficiency to manipulate trigonometric functions.

Finally, with the documentations of related studies presented herein, the researcher concluded that the findings of this investigation conformed with the results of the previous studies which disclosed that the learners' difficulties in integration are markedly attributed to the inherent mathematical knowledge and skills acquired by the students from basic mathematics in high school and in college.

5. Recommendations

The findings of this study may encourage the learners to recognize and understand the nature of their difficulties in indefinite integrals and know the reasons underlying their struggle. They should value analyzing their errors for diagnosis and remediation; take advantage of such errors as learning opportunities to understand the subject matter; and choose the most helpful learning strategies. They should spend more time practicing integration using different formulas and techniques involving trigonometric functions to enhance their skills in dealing with such functions. They have to consider the relational understanding of calculating indefinite integrals from the knowledge that they acquired from their mathematics foundation subjects such as algebra and trigonometry to advanced mathematics courses like differential and integral calculus.

The results of the investigation suggest the need to maintain the highest standard of instruction for teachers handling mathematics subjects, especially on the pre-requisite subjects such as trigonometry, for this serve as the foundation of students in higher mathematics courses. Calculus teachers should look upon the background knowledge and skills of the learners prior to the introduction of a more complex topic of mathematics. They may also consider the relevance of research literature to the teaching and learning of mathematics for such studies may provide insights as regards to the nature of the difficulties of their students. Varied classroom learning activities should be employed for these could lead to the identification and analysis of their students' difficulties in the subject matter. Intensive interventions should be offered to address specific areas of weaknesses of the learners and the areas of strengths should be recognized to deepen the students' understanding of the topic. The process of continuous diagnosis and customized classroom instructions should be carefully planned until such time that the learners overcome their difficulties.

May this study encourage future researchers to conduct further investigations to a larger number of participants covering more variables related to students' learning of advanced courses in mathematics.

References

Berggren, J. L. (2016). Calculus: Mathematics. Encyclopedia Britannica. Retrieved from http://www.britannica.com/topic/calculus-mathematics

Byers, P. (2010). Investigating Trigonometric Representations in the Transition to College Mathematics. The College Quarterly, Seneca College of Applied Arts and Technology, 13 (2).

Creswell, J. W. (2014). Research Design: Qualitative, Quantitative and Mixed Methods Approaches, Fourth Edition, Sage Publications, Inc., 269-273.

De Villiers, M. & Jugmohan, J. (2012). Learners' Conceptualization of the Sine Function during an Introductory Activity Using Sketchpad at Grade 10 Level. Educ. Matem. Pesq., São Paulo, 14 (1), 9-30.

Fui Fui, L. & Shahrill, M. (2013). Investigating Students' Conceptual Knowledge and Procedural Skills in Trigonometry. Australian Association for Research in Education. Retrieved from http://www.aare.edu.au/publications-database.php

Gur, H. (2009). Trigonometry Learning. New Horizons in Education, 57 (1), 67-80.

Hirst, K. E. Classifying Students' Mistakes in Calculus. Retrieved from http://www.math.uoc.gr/~ictm2/Proceedings/pap31.pdf

Huang, C. H. (2015). Calculus Students' Visual Thinking of Definite Integral. American Journal of Educational Research, 3 (4), 476-482.

Kiat, S. E. (2005). Analysis of Students' Difficulties in Solving Integration Problems. The Mathematics Educator, 9 (1), 39-59.

Leithold, L. (1996). The Calculus 7, Harpercollins College Division.

Lithner, J. (2011). University Mathematics Students' Learning Difficulties. Education Inquiry, 2 (2), 289-303.

Maharaj, A. (2008). Some Insights From Research Literature for Teaching and Learning Mathematics. South African Journal of Education, 28, 401-414.

Mahir, N. (2009). Conceptual and Procedural Performance of Undergraduate Students in Integration. International Journal of Mathematical Education in Science and Technology, 40 (2), 201-211.

Metaxas, N. (2007). Difficulties on Understanding the Indefinite Integral. In Woo, J. H., Lew, H. C., Park, K. S. & Seo, D. Y. (Eds.). Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, 3, 265-272.

Mikula, B. D. & Heckler, A. F. (2013). Student Difficulties With Trigonometric Vector Components Persist In Multiple Student Populations. PERC Proceedings, American Association of Physics Teachers, 253-256.

Mokhtar, Mohd, Tarmizi Rohani et al. (2010). Enhancing Calculus Learning Engineering Students Through Problem-based Learning. WSEAS Transactions on Advances in Engineering Education, 7 (8).

Moskal, B. M. (2002). Calculus: Mathematics. Encyclopedia.com. Retrieved from http://www.encyclopedia.com/ topic/calculus.aspx

Orton, A. (1983). Students' Understanding of Differentiation. Educational Studies in Mathematics,14, 235-250.

Orton, A. (1983). Students' Understanding of Integration. Educational Studies in Mathematics, 14, 1-18.

Pepper, R. E., Chasteen, S. V., Pollock, S. J., & Perkins, K. K. (2012). Observations on Student Difficulties with Mathematics in Upper-division Electricity and Magnetism. Physical Review Special Topics – Physics Education Research 8, 010111, 1-15.

Peterson, T. S. (1968). Calculus with Analytic Geometry, A Harper International Edition.

Salazar, D. A. (2014). Salazar's Grouping Method: Effects on Students' Achievement in Integral Calculus. Journal of Education and Practice, 5 (15), 119-126.

Siyepu, S. W. (2015). Analysis of Errors in Derivatives of Trigonometric Functions. International Journal of STEM Education, 2 (16), 1-16.

Souto, B. & Gomez-Chacon I. (2011). Challenges with Visualization: The Concept of Integral with Undergraduate Students. Proceedings The Seventh Congress of the European Society for Research in Mathematics Education (CERME-7), University of Rzeszow, Poland.

Stewart, J. (2010). Calculus, Seventh Edition, Cengage Learning.

Tall, D. (1993). Students' Difficulties in Calculus. Proceedings of Working Group 3, ICME-7, Quebec, Canada, 13-28.

Usman, A. I. (2012). Analysis of Algebraic Errors in Applied Calculus Problem Solving. 12th International Congress on Mathematical Education, COEX, Seoul, Korea.

Weber, K. (2008). Teaching Trigonometric Functions: Lessons Learned from Research. Mathematics Teacher, 102 (2), 144-150.

Yee, N. K. & Lam, T. T. (2008). Pre-University Students' Errors in Integration of Rational Functions and Implications for Classroom Teaching. Journal of Science and Mathematics Pre-University Students' Errors Education in Southeast Asia, 31 (2), 100-116.

Zakaria, E. & Salleh, T. S. (2015). Using Technology in Learning Integral Calculus. Mediterranean Journal of Social Sciences. 6 (5), 144-148.