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LESSONS IN GRADE 11 MATHEMATICS USING GRAPHING CALCULATOR APPLICATIONS

Cristian Mark B. Balantes

Graduate Student, Bicol University, Legazpi City, Philippines
cristianmark25@gmail.com

Daves L. Tonga

Professor, Bicol University, Legazpi City, Philippines
dltonga@bicol-u.edu.ph

Abstract

Since mobile devices are becoming ubiquitous in today's student generation and to replace bulky and costly handheld graphing calculators, this quasi-experimental study determined the effects of the lessons using graphing calculator applications in a Mathematics classroom. The study involved two intact classes of Grade 11 GAS students of San Jose National High School, Malilipot, Albay. The experimental group was taught lessons using graphing calculator applications with the integration of contextualized, inquiry-based, collaborative and interactive learning approaches. On the other hand, the control group received the same lessons but without the use of applications. The actual conduct of the lessons was video-recorded and observed by the teacher-observers to determine and analyze carefully the students' notable behaviors. Further, the effects of the lessons in terms of students' performance, mental effort, and interest were assessed by using teacher-made test, Paas Mental Effort Rating (PMER) scale, and Mathematics Interest Inventory, respectively. Students' journal entries, teachers' observation notes, and focus group discussions' responses were used to triangulate the results. Qualitative and quantitative analyses showed that the lessons using graphing calculator

applications supported student collaboration and facilitated better teacher-student interaction. It also allowed students to become comfortable when and how to use the applications, as well as making them active and participative during learning activities. It further enhanced students' conceptual understanding and process skills, and increased their level of interest towards Mathematics. The use of the applications also afforded the minimizing of students' extraneous cognitive load by investing lower mental effort in learning the lessons. To this extent, the use of graphing calculator applications in Mathematics lessons is needed for the betterment of learning the subject.

Keywords

Lessons, Graphing Calculator Applications, Learning Approaches, Behavior, Effects

1. Introduction

Mathematics is viewed by many students as one of the most challenging subjects, which they would not undertake if they had an option. To address these preconceived notions and to develop students' interest and motivation in learning Mathematics, educators have come up with various innovative techniques that teachers can adopt in their Mathematics classrooms. One such technique is the incorporation of technological tools like graphing calculators (Nzuki, 2016).

Graphing calculators offered many benefits in the classroom, such as its being used to develop concepts, drills, enhance problem solving, and most importantly save time. However, because of its very high price, it became inaccessible to most of the students in the Philippines. Now, since smartphones and tablets are becoming ubiquitous in today's student generation and that it can fit in anyone's pocket, anytime, anywhere, students can already experience the features of a graphing calculator as a mobile application in their devices. This mobile application of a graphing calculator is what we called "graphing calculator application".

With that, this study aimed to develop lessons in Grade 11 Mathematics using graphing calculator applications. The developed lessons integrated at least three features. These features were contextualized, inquiry-based, collaborative, and interactive learning approaches. It also sought to determine the notable behaviors of the students observed during the conduct of the lessons, as well as its effects in terms of students' conceptual understanding, process skills, cognitive load (mental effort), and interest towards Mathematics.

The developed lessons were also conducted in the Senior High School department of San Jose National High School, Malilipot, Albay, 1st Semester, School Year 2018-2019. Likewise, the graphing

calculator applications used were *Graphing Calculator by Mathlab*, *Free GraphCalc 2*, *Graphing Calc (GeoGebra)*, and *Desmos*. All of these applications run in android platforms or tablets with an operating system equal to 4.0 and above, and are free in trusted and selected app stores.

2. Materials and Methods

This study utilized a quasi-experimental design. It consisted of experimental group and control group, wherein, the experimental group received lessons using graphing calculator applications and the control group was taught same lessons but without the use of the applications. The Grade 11 GAS A was randomly selected as the experimental group, while the Grade 11 GAS C as control group. This was done by random-sampling using draw lots of the three sections in Grade 11 GAS level. The classes were also grouped heterogeneously during the enrolment period. Further, the conduct of this study was divided into three phases: (1) Planning and Initial Evaluation Phase, (2) Implementation Phase, and (3) Final Evaluation Phase.

The planning and initial evaluation phase involved the development of the lesson plans. The lesson plans have three parts: learning objectives, learning tasks, and instructional procedures. During the instructional procedures, the lesson plans followed the 7E's instructional model, which includes elicit, engage, explore, explain, elaborate, evaluate, and extend phases. The lesson plans were also evaluated and content validated by 16 Senior High School Mathematics teachers. This phase also included the preparation and validation of the research instruments. The research instruments used in the study are teacher-made test, Paas Mental Effort Rating (PMER) scale, Mathematics Interest Inventory, observation guides, journal writing guides and focus group discussion guides.

The next phase involved the implementation phase. The researcher was the one who taught both the experimental and control groups. Both classes were being video-recorded and observed by teachers to document the day-to-day implementation. In addition, the control group met at 7:30-8:30 a.m. and the experimental group at 9:45-10:45 a.m. The elapsed time on the schedule of the two classes may have caused a possible contamination and threat to the internal validity of the study. The applications could be easily transferred and that the experimental group could access the notes of the control group. However, the researcher firmly declared that no such things happened during the course of the study. After all the discussion of the developed lessons, the teacher-made test and interest inventory were administered as post-test during the vacant period of both groups. The focus group discussion for the students and for the teachers were also executed.

The final phase of the study is the final evaluation phase through quantitative and qualitative analyses of the gathered data. The statistical treatments involved mean, weighted mean, performance level, mean gain, *F*-test, *t*-test for dependent and independent samples, coefficient of determination, and effect size (Cohen’s *d*: standardized mean difference). The level of significance used is at $\alpha = 0.05$.

3. Results and Discussion

3.1 Developed Lessons and their Features

Eight lessons using graphing calculator applications were developed to cater to the learning competencies in Grade 11 General Mathematics. Table 1 presents the learning competencies and topics of the developed lessons. The graphing calculator applications that were used and how it was utilized during the activities in the lessons were also included.

Table 1: *Lessons in Grade 11 Mathematics using Graphing Calculator Applications*

Learning Competency	Lesson No. and Title	Graphing Calculator Applications Used	Activities using Graphing Calculator Applications
<ul style="list-style-type: none"> represents real-life situations using rational functions 	1. Representations of Rational Functions (Part 1)	Graphing Calculator by Mathlab, Free GraphCalc 2	<ul style="list-style-type: none"> constructing table of values graphing rational functions
<ul style="list-style-type: none"> represents a rational function through its table of values, graph, and equation 	2. Representations of Rational Functions (Part 2)	Graphing Calculator by Mathlab, Free GraphCalc 2	<ul style="list-style-type: none"> constructing table of values graphing rational functions
<ul style="list-style-type: none"> finds the domain and range of a rational function determines the intercepts, zeroes, and asymptotes of rational functions 	3. Domain, Intercepts and Zeroes of Rational Functions	Graphing Calculator by Mathlab, Free GraphCalc 2, Graphing Calc (GeoGebra), Desmos	<ul style="list-style-type: none"> constructing table of values graphing rational functions checking for the intercepts and zeroes of rational functions factoring quadratic expression
<ul style="list-style-type: none"> graphs rational functions 	4. Vertical and Horizontal Asymptotes of Rational Functions	Graphing Calculator by Mathlab, Graphing Calc (GeoGebra)	<ul style="list-style-type: none"> evaluating rational functions constructing table of values graphing rational functions finding the vertical and horizontal asymptotes
	5. Graphs of Rational Functions (Part 1)	Graphing Calc (GeoGebra), Desmos	<ul style="list-style-type: none"> graphing rational functions together with its asymptotes checking for the formulated

Learning Competency	Lesson No. and Title	Graphing Calculator Applications Used	Activities using Graphing Calculator Applications
			rational function that might have the given graph
	6. Graphs of Rational Functions (Part 2)	Graphing Calc (GeoGebra), Desmos	<ul style="list-style-type: none"> graphing rational functions together with its asymptotes checking for the formulated rational function that might have the given graph
	7. Range of Rational Functions	Graphing Calc (GeoGebra), Desmos	<ul style="list-style-type: none"> graphing rational functions together with its asymptotes locating the extremum (maximum and minimum values) of a rational function
<ul style="list-style-type: none"> solves problems involving rational functions 	8. Solving Real-life Problems involving Rational Functions	Graphing Calculator by Mathlab, Graphing Calc (GeoGebra), Desmos	<ul style="list-style-type: none"> evaluating rational functions graphing rational function together with its asymptotes locating the extremum (maximum and minimum values) of a rational function

Furthermore, the developed lessons in Grade 11 Mathematics using graphing calculator applications integrated at least three features. These features include contextualized, inquiry-based, collaborative, and interactive learning approaches.

3.1.1 Contextualized Learning Approach

Contextualized learning approach built on the recognition that students learn more effectively when they are taught in a hands-on, real-world context rather than in an abstract manner (Baker, Hope & Karandjeff, 2009). Two of the eight lessons were integrated with contextualized learning approach. These lessons are lessons 1 and 8. The two lessons provide diverse real-life situations that are related to the environment and/or experience of the learners.

In Lesson 1, the students represented real-life situations using rational functions. This was done by formulating a rational function based on the context of the problem, constructing table of values and graphing the rational function. The table of values and graphs are also restricted only for zero and positive values since the real-life situations given were restricted to these values. Meanwhile, in Lesson 8, the students solved real-life problems involving rational functions. The real-life situations included were about speed and time, perimeter of a rectangular vegetable farm, concentration of the

drug in the bloodstream, cost (in pesos) of removing smokestack pollutants, and the average cost per product to produce.

3.1.2 Inquiry-based Learning Approach

Inquiry-based learning is an approach that invites students to explore academic content by posing, investigating, and answering questions (Towns & Sweetland, 2008). Two levels of inquiry were used in this study. These were structured inquiry and guided inquiry. In structured inquiry, the teacher provides the questions and procedures; however, the students are the ones to generate explanations. This type of inquiry was used for Lesson 5 and 6, which were all about graphs of rational functions. While, the remaining lessons incorporated guided inquiry. In this level of inquiry, the teacher provides students with the questions that will guide them for the discovery of the mathematical concepts and skills involved in the developed lessons.

Aside from the two levels of inquiry, the questions in the activities were also anchored to more specific approaches that are in-line with inquiry-based learning. These included problem-based, discovery, and cooperative learning. Problem-based learning is an approach in which students work collaboratively in groups to solve problems given. Lessons 1 and 8, where real-life situations are given, were the lessons integrated with problem-based learning.

Additionally, discovery learning engages the learners to draw their own conclusions based on the instructions and guide questions on the tasks given to them. To achieve discovery of concepts and skills involved in the lesson, students must link their prior knowledge to the new learning upon the discussions (Balim, 2009). It also allows the students to create meaningful ways on how to explain a concept and to solve problem genuinely. Discovery learning was used specifically in Lessons 3, 4, and 7. These lessons tackled the different properties of rational functions, i.e. domain, intercepts, asymptotes, and range, and incurred the students to discover the different ways of finding these properties.

Lastly, cooperative learning is involved in all the developed lessons. It utilized groups so that students work together to maximize their own and each other's learning (Bishnoi, 2017). Before the group activities proceed to individual learning, the teacher will ask the students how they feel working with their classmates and if they are cooperative. In this way, the teacher can assure that the cooperation between the students really take place, through verbal response of the students, aside from the fact that he will constantly monitor the students during group activities. In addition, cooperative learning worked hand-in-hand with collaborative learning features of the developed lessons.

3.1.3 Collaborative Learning Approach

The main purpose of integrating collaborative learning in the lessons is to compensate those students who do not own a smartphone/tablet. By working collaboratively, each student shall engage in the lessons and gauge with the mathematical concepts and skills even if he/she cannot provide for himself/herself a smartphone/tablet.

All lesson incorporated collaborative learning approach. For every lesson, the students were divided into groups. To achieve active participation among the students, specific roles were assigned to each member of the group. These roles included being a leader, secretary, recorder, materials manager, peacekeeper, speaker, and timekeeper. Although each student has his/her own responsibilities, the group must see to it that each member will collaborate by sharing and brainstorming his/her ideas. Additionally, different group activities were provided in each lesson like “layered cake” discussion and team competition. These activities helped the students to work together to search for understanding or solutions in a particular learning task.

3.1.4 Interactive Learning Approach

With this learning approach, students are invited to participate in the conversation, through technology or through group exercises in class (Jony, 2016). All of the lessons integrated this approach. For each lesson, the learning activities provide greater opportunity for an active interaction between the students, students and the teacher, students and the graphing calculator applications, and students and the learning activities.

From the elicit-phase to evaluate-phase, the lessons helped the students to sharpen critical thinking skills with the aid of different learning activities and the use of graphing calculator applications. The lessons also taught the students on how to work successfully in groups as well as creating strategies on how to learn the topic not just by memorizing information.

3.2 Notable Behaviors

Notable behavior is defined in this study as the notable actions, reactions and response of the students, which are observed during the conduct of the lessons. Table 2 presents the classification of the notable behaviors executed by the students that were observed by the lesson observers. The classifications were patterned based on the types of interaction, which include student-to-student, student-to-teacher, student-to-graphing calculator applications, and student-to-learning activities.

Table 2: Classifications of Notable Behaviors Observed during the Conduct of the Lessons

Student-to-Student	Student-to-Teacher	Student-to-Graphing Calculator Applications	Student-to-Learning Activities
<ul style="list-style-type: none"> • Cooperation and collaboration • Accountability to each responsibility • Kindness to other students 	<ul style="list-style-type: none"> • Listening attentively and following instructions • Raising hand to answer question or when asking clarifications 	<ul style="list-style-type: none"> • Curiosity • Eagerness and confidence • Interest and motivation • Following rules about smartphones/ tablets 	<ul style="list-style-type: none"> • Prepared and unprepared • Academic noise • Following the rules of the activities Staying on task • Being active and participative

3.2.1 Student-to-Student Behavior

The lessons using graphing calculator applications consisted of learning activities that enable students to take a more participatory and active role in their learning. During group activities, the observers always noticed that the students were working cooperatively and collaboratively on their tasks. The assigned leaders of the groups, who were typically the fast learners, were the ones who guided and helped their classmates to perform the given tasks. In addition, it is observed that the group had a brainstorming and were not only dependent on the leader. Every member of the group shared and gave his/her ideas and inputs so that they could finish the activities correctly and ahead of time. The left photo of Figure 1 shows that the group worked cooperatively by letting the other two students work on the given task as well as the other members of the group. After they answered the activity sheet, the group then synthesized all their findings in order to come up with a final output and present it to the class.



Figure 1: Students Working Cooperatively and Collaboratively in Performing a Task (left) and Students Showing Accountability on their Responsibility (right)

It was also imposed in the class that every time there were groupings, specific roles were assigned to each member of the group. For this reason, each student was accountable to his/her responsibility. For example, in Lesson 2, an observer wrote, *“Each group divides the tasks during group activities, e.g., someone will construct the table of values on the app, some will write it in the cardboard, some will prepare the Cartesian plane, and the others will plot and construct the graph.”* Doing such, each group finished the task ahead of time and that each member had performed his/her responsibility in his/her group. In addition, the right photo of Figure 1 depicts students’ accountability for their responsibility. It can be seen that the materials manager was handling the mobile device so that the secretary was able to write the answers in a comfortable way. This made the group easily facilitate the activities or tasks given to them.

Moreover, it was also observed that those students who owned a smartphone/tablet were very kind to lend their gadgets to their other classmates. In some instances, when the group had an ample number of smartphones, they let the other groups borrow some of them, especially when that group did not meet the requirements of a 1:2 or 1:3 ratio with gadgets.

3.2.2 Student-to-Teacher Behavior

All throughout the course of the conduct on the lessons, listening attentively and following instructions were always manifested by the students while they are interacting with the teacher. This was based on the observation notes of an observer who wrote, *“The students interact well with their teacher by listening attentively and following his instructions.”* When the teacher was the one who was talking to the class, all the students were listening attentively, except if they were a little bit distracted. In addition, the instructions of the teacher, most especially when to use graphing calculator applications and to return all the materials used, were being followed by the students.

The observers also cited that whenever the teacher asked questions, students raised their hand to answer and to be called. In some cases, they answered in chorus, but as much as possible answering questions and explaining their work were done by raising their hands. Additionally, students also raised their hands when they had some misconceptions or confusion about the topics. The teacher let the other students make the clarifications, but if it entailed deeper explanations, the teacher was the one who was clarifying those misconceptions.

3.2.3 Student-to-Graphing Calculator Applications Behavior

Since graphing calculator applications were new to most of the students during the first day of implementation, an observer cited on his observation notes, *“The students like the applications since it*

makes their work easier. Of course, some of the students find it hard to use the applications because they didn't know yet how to operate it, but on the second day of the discussion, they already used to it. In addition, some became curious about the applications and explored their features." The observers also indicated that this also happened everytime a new feature of the applications was being introduced to the class. For instance, when the teacher introduced how to graph the asymptotes of a rational function in Graphing Calc (GeoGebra), the students became curious and listened to the teacher on the procedures on how to do it.

Further, since most of the learning activities were under time-pressure, the students were very eager to use immediately their graphing calculator applications in order to perform their tasks ahead of time. In most cases, it was noticed that the students were also eager to answer the questions of the students since they had a device that helped them tell whether their answer was correct. They also added that the graphing calculator applications made the students confident and proud of their answers and were not shy in sharing it with the class.

As the students became adept in using the graphing calculator applications, they also became more excited and motivated in performing the learning activities using the applications. The students were very interested to perform the tasks since they found it easy because of the applications. During the lessons on sketching the graphs of rational functions, the students were not yet allowed to use graphing calculator applications so that they would also know the manual process of graphing. However, when they saw that their sketch of the graph was correct in the graphing calculator applications, they were so happy and became more motivated to try another one. The observers then believed that this positive attitude towards the graphing calculator applications could help the students improve their performance especially in sketching different kind of graphs.

As the observers reviewed the videos of the actual conduct of the lessons, they found out and identified that were few students attempted to use their mobile devices for texting, messaging, chatting or doing other things that were not related to the lesson. However, since the teacher constantly monitored the class during activities, and the leaders of each group also supervised their members, violating this rule was avoided.

3.2.4 Student-to-Learning Activities Behavior

Almost all of the students were prepared for the discussion of each lesson. They already answered their assignment and were ready to be checked or passed during the class. The students already charged their mobile phones that should not have a low battery level. However, the observers

also identified unprepared students. These students were those that find the assignment or the past lesson hard. Some of them just copied the preparation of their classmates not asking how it was done.

The observers also stated that due to the eagerness and enthusiasm of the students to perform the different activities, which were under time-pressure, making noise was not avoided. Yet, they classified it as academic noise, which was a positive behavior inside the classroom. They also added that it was good that noise was minimized in order not to disturb other classes.

Like in using graphing calculator applications, the students also followed the rules of the different activities. It was noted that there were some students asking about the rules, which is a positive behavior in order for them to be totally engaged in the activities. Meanwhile, the lessons comprised various learning activities that entailed the students to be more active. Although there were transitions of activities that happened in most of the lessons, the students were able to stay on task and still performed succeeding activities actively.

During the last two lessons, the observers also stated in their observation notes that *“I noticed that those students who were too shy to recite are now confident to explain their answers to the class.”* This means that those shy students observed during the first few lessons are starting to raise their hands and eventually volunteer themselves to answer questions or to present their outputs.

Generally, the researcher and the observers cited that the students were active, involved, and participative all throughout the conduct of the lessons. It is important to note that using technology is not only intended for content delivery, but for on-task student behavior as well. Integrating technology in the classrooms can increase student engagement and motivation to learning, allows for better teacher-student interaction, supports student collaboration, and assists in the accuracy of mathematical computation during learning activities. It also allows students to become more aware and comfortable in not only knowing how to use the technology, but also gives the students more confidence in the subject (Murphy, 2016).

3.3 Effects

3.3.1 Conceptual Understanding

By allowing the students to use graphing calculator applications during lessons, they were able to grasp the different mathematical concepts and relationships about rational functions through investigations and explorations. Table 3 presents the statistical data gathered from the pre-test and post-test of the experimental and control groups along conceptual understanding.

Table 3: Conceptual Understanding Test Results Summary Statistics

Topics	EXPERIMENTAL GROUP						CONTROL GROUP					
	Pre-test			Post-test			Pre-test			Post-test		
	Mean	PL		Mean	PL		Mean	PL		Mean	PL	
		%	D		%	D		%	D		%	D
Representations	1.19	40	LM	2.30	77	M	1.04	35	LM	1.87	62	NM
Domain & Intercepts	1.55	39	LM	2.96	74	NM	1.35	34	LM	2.59	65	NM
Asymptotes	1.21	24	NoM	3.74	75	M	1.35	27	LM	3.35	67	NM
Graphs	1.38	23	NoM	4.64	77	M	1.59	27	LM	3.78	63	NM
Range	0.51	26	LM	1.64	82	M	0.65	33	LM	1.43	72	NM
Total	5.85	29	LM	15.30	77	M	5.98	30	LM	13.02	65	NM
Mean Gain	+9.45						+7.04					
r^2	0.67						0.34					
Effect Size	0.97 (Very Large Effect)											

Legend: PL – Performance Level D – Mastery Level Descriptive Equivalence
 NoM – No Mastery NM – Near Mastery LM – Low Mastery M – Mastery

It is shown in Table 3 that in the pre-test, the experimental and control group had a mean score of 5.85 and 5.98, respectively, which both indicates low mastery level. Through two-tailed *F*-test for testing the difference between two variances ($p = 0.29$), an independent two-tailed *t*-test with equal variances was employed between the pre-test scores of the experimental group and group. It resulted to a *p*-value of 0.62 that indicates no significant difference between their pre-test scores. This denotes that the two groups had the same level prior to the implementation of the study. This further signifies that the heterogeneity of the students was established and the two groups were comparable in terms of conceptual understanding.

In the post-test, the experimental group had a mean score of 15.30, which is an increase from 5.85 in the pre-test and corresponds to a mean gain of 9.45. These results were further analyzed using a dependent right-tailed *t*-test. The generated *p*-value of 1.30176×10^{-26} indicates that the increase in the scores of experimental group was highly significant. This entails that the students enhanced their conceptual understanding after the conduct of the lessons. On the other hand, the performance level of the experimental group was increased to 77% (mastery) from 29% (low mastery). This means that the students mastered the different mathematical concepts about rational functions. Further, 67% of the variation in the post-test results of the experimental group was accounted for by its linear relationship with the pre-test results. This implies that more students in the said group showed very close spread of scores.

To substantiate the results, sample students journal entries were taken. For instance, when they were asked how the use of graphing calculator applications helped them to understand better the concept of asymptotes, one wrote that *“Of course yes. Because of the applications, I understand better the concepts of asymptotes. We knew that an asymptote is a line to which the graph of the function gets closer and closer but will never touches it. At first, you will think that it already touched the graph, but if you will drag and zoom in the scree, it can be seen that it would not.”* These responses revealed that the students had better understanding on concept of asymptotes with the help of graphing calculator applications. Rather than confining the students to the mathematical idea of asymptote of a function, graphing calculator applications provided them the opportunity to visualize and explore more this mathematical concept by simply dragging and zooming in the screen in the application. When the vertical asymptote and a portion of the graph seemed to touch each other, the students had to zoom in the screen of their mobile devices to see clearly that the two certainly never intersect each other.

Meanwhile, Table 3 also indicates that the control group had a mean score of 13.02 in the post-test, which is an increase of 7.04 from 5.96 in the pre-test. The results were further analyzed using a dependent right-tailed *t*-test. The *p*-value is 1.25394×10^{-23} indicating that the increase in the scores of control group was highly significant. The performance level of the control group had also increased from 30% (low mastery) to 67% (near mastery). Moreover, the computed shared variation of control group is only 34%.

The results of the pre-test and post-test of both experimental and control groups suggest that the students' conceptual understanding was improved. However, comparing their post-test scores, the mean of the experimental group, which is 15.30, is higher than the mean of the control group, which is 13.02. The *F*-test result ($p = 0.03$) allows the use of an independent right-tailed *t*-test with equal variances. The *t*-test resulted in a *p*-value of 3.6656×10^{-5} that indicates a highly significant difference between the post-test scores of the experimental and control groups. Likewise, the experimental group mastered (77%) almost all the topics included in the lessons, while the control group was in near mastery level (65%). Further analysis also shows that there was a very large effect of the lessons using graphing calculator application. This was based from the effect size of 0.97. From these results, it is established that the lessons using graphing calculator applications are better in enhancing the conceptual understanding of the students than those lessons without it. In connection with the focus group discussion for teachers, they stated, *“The experimental group has the chance for deepening of*

concepts since the graphing calculator apps probe the analysis of those concepts. In addition, that is unlikely to happen in the control group due to the fact that they don't have such device.”

The aforementioned findings showed that the students enhanced their conceptual understanding after the conduct of the lessons using graphing calculator applications. This was occurred by producing clear visualization and simplification of the concepts, as well as providing the students the chance to explore, compare and contrast different mathematical relationships. These findings are consistent with the study of Ye (2009) about the use of graphing calculators in secondary Mathematics. The use of graphing calculators does not affect Mathematics learning, but helps the students to understand concepts and promotes changes for the traditional learning approach to diverse learning through self-learning, exploration, and cooperation.

3.3.2 Process Skills

The process skills involved in this study are representing, connecting, reasoning, and precision. Table 4 shows the process skills test results summary statistics.

Table 4: Process Skills Test Results Summary Statistics

Skills	EXPERIMENTAL GROUP						CONTROL GROUP					
	Pre-test			Post-test			Pre-test			Post-test		
	Mean	PL		Mean	PL		Mean	PL		Mean	PL	
%		D	%		D	%		D	%		D	
Representing	1.49	12	NoM	9.68	81	M	1.35	11	NoM	9.24	77	M
Connecting	1.94	28	LM	5.81	83	NFM	1.65	24	NoM	4.72	67	NM
Reasoning	1.60	6	NoM	20.40	76	M	1.83	7	NoM	16.83	62	NM
Precision	1.51	10	NoM	12.32	82	M	1.59	11	NoM	8.67	58	NM
Total	6.53	11	NoM	48.21	79	M	6.41	11	NoM	39.46	65	NM
Mean Gain	+41.68						+33.05					
r^2	0.53						0.28					
Effect Size	1.13 (Huge Effect)											

Legend: PL – Performance Level D – Mastery Level Descriptive Equivalence
 NoM – No Mastery NM – Near Mastery NFM – Near Full Mastery
 LM – Low Mastery M – Mastery

Prior to the intervention, both the experimental and control groups are in no mastery level based on their pre-test scores of 6.53 and 6.41, respectively. Since the *F*-test between the pre-test scores of the two groups was resulted to $p = 0.48$, an independent two-tailed *t*-test with equal variances was then used. It ensued to a *p*-value of 0.82, which signifies no significant difference between the pre-test scores of the experimental and control groups. This further indicates that the

heterogeneity of the students was established and the two groups were comparable along process skills.

Meanwhile, the experimental group had a mean score of 48.21 in the post-test, which corresponds to a mean gain of 41.68 from the pre-test. Using a dependent t -test, it gives a p -value of 5.28619×10^{-35} indicating that the increase in the scores of experimental group was highly significant. This entails that the students improved their process skills after the conduct of the lessons. Additionally, 53% of the variation in the post-test scores is accounted for by a linear relationship with the pre-test scores.

It is observed that the experimental group's connecting skills is at near full mastery level. Seeing and exploring the graphs help the students to reinforce the abstract concepts and to connect the algebraic and graphical process of finding the properties of a function. For example, the students must connect the concepts and manual procedures of finding the x and y intercepts of a rational function to the graphical way of locating them. The students then used a graphing calculator application to graph such rational function then tap on the x -axis and y -axis to determine the x and y intercepts of the function, respectively.

Next to connecting skills, the highest performance level obtained by the experimental group was in their precision skills. The group had a performance level of 79% (mastery) in the post-test, which is an increase from 11% (no mastery) in the pre-test. In this study, precision skills mainly deal with graphing rational functions. From the very first lesson, the experimental group already used graphing calculator applications to graph rational functions to explore its properties, not actually graphing it. This implies that the student had already visualized what the graphs of rational functions look like. During the discussion about graphs of rational functions, the students sketched first the graph of a rational function manually by following the step-by-step procedures. After which, they used a graphing calculator application to check whether their sketch of graph was correct or not. This helped them to compare their sketch to the actual graph on the application, enabling them to identify in what step they were mistaken. Through this, they rectified the process of that step, thus, improving their sketch of the graph by not just copying the graph displayed on the application.

The learning activities in the lessons also provided the experimental group the chance to practice their precision skills by graphing different rational functions by hand, using the applications or the combination of the two. The main purpose of learning how to graph by hand was to develop the skills of the students in following the step-by-step procedures in graphing rational functions. However,

since they were allowed to use graphing calculator applications to graph such function, they were able to check whether their manual sketch of the graph was properly constructed or not. Thus, when the students were asked if using graphing calculator applications improved their manual graphing of rational functions, the students stated that graphing calculator applications were not only a tool for checking, but it also aided them to determine what should be the appropriate scaling of axes when graphing rational functions. They further asserted that they became adept in graphing manually by creating visual imagination on the different kinds of graphs of rational functions. For example, if the rational function has two vertical asymptotes, the students had the assurance that the graph would split into three different curves.

Furthermore, the experimental group had mastered both the representing and reasoning skills. This means that the use of graphing calculator applications during the lessons had a positive impact on how the students represent real-life situations using rational functions and on how they can provide mathematical solutions or explanations for a certain problem. For instance, in order to find the horizontal asymptote of a rational function, students answered activity sheets that contain guided questions and with the help of the applications, wherein the horizontal asymptote can be automatically graphed, they come up with the necessary conclusions. The students then used these conclusions to justify their answers whenever another related problem is given to them. These results were related with the study of Spinato (2011) indicating that graphing calculators had a positive impact upon students' reasoning skills and were most effective in the areas of initiating a strategy and monitoring progress.

Since the study also involved a control group, their pre-test and post-test scores were also subjected to analysis to indicate that learning also took place in them. As indicated in Table 4, the group had a mean score of 39.46 in the post-test, which is an increase of 33.05 from 6.41 in the pre-test. By means of *t*-test, a *p*-value ensued to 2.53989×10^{-31} suggesting that the increase in the scores of control group was also highly significant. However, computed coefficient of determination (28%) is very low.

The pre-test and post-test results of both experimental and control groups suggest that the students' process skills were developed and improved. However, comparing their post-test scores, the mean of the experimental group, which is 48.21, is higher than the mean of the control group, which is 39.46. Likewise the result of the two-tailed *F*-test between the post-test scores of the two groups allowed the use of independent right-tailed *t*-test with equal variances. The *t*-test resulted in a *p*-value

of 1.49137×10^{-7} that signifies a highly significant difference between the post-test scores of the experimental and control groups. Computation of effect size revealed that there was a huge effect (1.13) of the intervention. This implies that the lessons using graphing calculator application are much better in improving the process skills of the students.

3.3.3 Cognitive Load (Mental Effort)

The Paas Mental Effort Rating (PMER) scale was used to determine the mental effort invested by the students in learning a particular lesson. It is shown in Table 6 the mental effort invested by the experimental and control groups in learning each lesson about rational functions.

Table 5: Mental Effort Invested in Learning Rational Functions

Topic	EXPERIMENTAL GROUP		CONTROL GROUP	
	Mean	Description	Mean	Description
Representations	3.45	Low mental effort	5.51	Very high mental effort
Domain and Intercepts	3.06	Low mental effort	5.46	High mental effort
Asymptotes	2.40	Very low mental effort	5.78	Very high mental effort
Graphs	3.41	Low mental effort	6.00	Very high mental effort
Range	3.21	Low mental effort	5.72	Very high mental effort
Real-life Problems	4.87	High mental effort	6.57	Very, very high mental effort
Total	3.41	Low mental effort	5.82	Very high mental effort

As reflected in Table 5, the experimental group invested low mental effort in learning the topics covered in the lessons while the control group invested very high mental effort. A *t*-test for unequal variances was conducted among the mean ratings and it ensued to a *p*-value of 5.62332×10^{-43} , which signifies that the difference between the mean ratings in terms of mental effort was highly significant. This implies that the experimental group who were taught lessons using graphing calculator applications had invested lower mental effort than the control group.

For deepening of the results, the learning activities using graphing calculator applications of the experimental group were elaborated and compared to those same learning activities in the control group. This helps to determine how the learning activities provided to each group affects the investment of mental effort in learning rational functions. In particular, in learning about graphs of rational function, the experimental group had invested low mental effort. They used the graphing calculator applications in order for them to check their sketch of graph. This gave them the opportunity to improve their sketch and to look for what was wrong or missing with it. In addition, the applications were used for the students to graph automatically the given rational functions in order to find its properties through the graphs. Thus, instead of consuming time to graph manually the rational

functions, the applications helped them to do the tasks and the students do the investigations. For the control group, learning about graphs of rational functions was done manually all throughout the lesson. This made them to invest very high mental effort in learning the topic.

The aforementioned results showed that the graphing calculator applications gave the students of experimental group the opportunity to visualize mathematical tasks, check the correctness of their answers obtained by analytical means, explore and discover different ways of solving problems, and solve laborious and non-routine problems inaccessible by algebraic techniques. These benefits would not have arisen without the use of graphing calculator applications and are in line with what Salomon et. al. (1991) termed effects with technology.

Additionally, reducing the cognitive load or mental effort for the students by taking over the mechanics of moving from one representation to another, frees the student to “focus on the connections between actions on the two systems.” The actions, which otherwise have a tendency to consume all of the student’s cognitive resources even before translation can be carried out (Nzuki, 2016; Kaput, 1992).

3.3.4 Interest

To determine whether the lessons in using graphing calculator applications could improve students’ interest towards Mathematics, the Mathematics Interest Inventory, was given as pre-test and post-test for both experimental and control groups.

Table 6: Mathematics Interest Inventory Results Summary Statistics

EXPERIMENTAL GROUP				CONTROL GROUP			
Pre-test		Post-test		Pre-test		Post-test	
Mean	D	Mean	D	Mean	D	Mean	D
4.05	MH	5.23	H	4.18	MH	4.39	MH

Legend: D – Level of Interest Description MH – Moderately High H – High

Statistical analysis on the pre-test mean ratings revealed that the experimental and control groups had the same level of interest towards Mathematics prior to the conduct of the study. This means that the two groups are comparable along this area. It is observed in Table 6 that after the conduct of the lessons using graphing calculator applications, the experimental group has a high level of interest towards Mathematics based on their mean rating of 5.23 in the post-test. This is an increase from 4.05 in the pre-test. Applying *t*-test, it resulted in a *p*-value of 7.00342×10^{-18} indicating a highly significant increase between the two mean ratings. This suggests that the experimental group had

significantly increased their interest towards Mathematics after the conduct of the lessons incorporating graphing calculator applications. On the other hand, the control group, which did not use graphing calculator applications during the lessons, had a moderately high level of interest based on their mean rating of 4.39 in the post-test, an increase from 4.18 in the pre-test. However, it is also interpreted as moderately high level of interest.

The data in Table 6 suggest that the interest for both groups was increased, however, the experimental group's level of interest had a highly significant increase, which is better than the significant increase of control group. This is supported by a *t*-test, which resulted in a *p*-value of 6.60464×10^{-12} implying highly significance. This result establishes that the lessons using graphing calculator applications are much better in increasing the level of interest of the students towards Mathematics as a subject. To substantiate these results, the experimental group stated that, *"Through the use of applications in the lessons, we are so interested to learn the subject because we became confident about our answers and that the apps helped us to perform the activities ahead of time."* Through the applications, the students became active and participative during the discussion and their confidence to recite was boosted since they knew that their answers were correct. This means that the hesitation of the students to recite that they might be embarrassed if their answer is wrong had significantly changed because of the applications.

It is also important to note that having a strategy that appropriates when and how to use the graphing calculator applications ensures that the technology does not take the place of, but rather enhances students' mathematical understanding and skills. Reciprocally, students should be able to use their mathematical knowledge to enhance their skills and strategies of using technology. As such, appropriate use of technology in the mathematical learning of students should foster the mutual reinforcement of mathematical and technological knowledge (Nzuki, 2011).

4. Conclusion

There were eight (8) developed lessons about rational functions integrating at least three features of contextualized, inquiry-based, collaborative, and interactive learning approaches. The notable behaviors of the students showed that the lessons using graphing calculator applications supported student collaboration and facilitated better teacher-student interaction. It also allowed students to become comfortable when and how to use the applications, as well as making them active and participative during learning activities. It further enhanced students' conceptual understanding and

process skills, reduced students' cognitive load or mental effort in learning the topics, and increased their level of interest towards Mathematics. Indeed, the use of graphing calculator applications in Mathematics lessons is needed for the betterment of learning the subject.

4.1 Research Limitations

This study mainly focused on the lessons in Grade 11 Mathematics using graphing calculator applications. The whole duration of the conduct of the study was from June to August 2018. The experimental group received lessons using graphing calculator applications with the integration of contextualized, inquiry-based, collaborative, and interactive learning approaches. The control group were taught the same lessons but without the use of graphing calculator applications. The control group was also allowed to use basic scientific calculators for their computations, which the researcher claimed does not affect the goal of the study. The researcher was also the one who taught the lessons for both groups.

Meanwhile, prior to the conduct of the study, the researcher e-mailed the developers of these graphing calculator applications if he could indicate the name of their applications in the study, and all of them agreed. More importantly, this study was an offline mobile learning intervention since Internet connection in the Philippines is not yet fast and reliable. Therefore, this is not a distance learning intervention.

4.2 Scope of Future Research

The researcher recommends for further research that the integration of graphing calculator applications in teaching Mathematics should have a 1:1 ratio with the students, as much as possible, to ensure greater participation among the students. Longer period of exposure on the use of graphing calculator applications is encouraged to effectively observe its effects on the mathematical understanding and skills of the students. Correlation between cognitive load and test scores can also be done to determine whether there is a relationship between the mental effort of a student in learning a topic to his or her test performance.

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