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TYPE-I ASSEMBLY LINE BALANCING WITH WORKLOAD SMOOTHING

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Abstract

Balancing the assembly lines plays an important role in increasing the productivity of the manufacturing systems. Depending on the objectives, there are various types of this problem. In this study, a Type-1 assembly line balancing problem is considered. It is known that the distribution of workload and equal distribution of idle time in station balancing is important for worker motivation and ergonomics. Once the number of stations is minimized, which is the primary objective in solving a Type-1 problem, the optimum solution found should be analyzed in terms of the smoothness index. Otherwise, the idle time can be distributed unevenly to the stations. Hence, one of the classical problems in assembly line balancing literature, Sawyer problem, is considered in this study. Firstly, the problem is solved via integer programming with Gurobi in Python. In the second stage, the line was smoothed via two techniques including the classical one and the *proposed min-max approach. Different cycle times are tested and a comparison is provided for the two techniques.*

Keywords

Assembly Line Balancing, Smoothness, Integer Programming, Workload

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1. Introduction

Assembly lines being the essence of discrete serial production systems play an important role in the efficiency of the manufacturing systems. Starting with the single-model assembly lines first seen at the Ford Highland Park Factory in 1913 (Wilson, 2013), they evolved into multi and mixed-model lines. In addition to the classification of assembly lines based on product type variety, they can also be categorized concerning layout types such as straight assembly lines (Salveson, 1955), parallel assembly lines (Süer and Dagli, 1994), u-shaped assembly lines (Miltenburg and Wijngaard, 1994) and two-sided assembly lines (Bartholdi 1993). Moreover, considering the objectives, an assembly line balancing problem is classified as Type-I if it seeks to minimize the number of stations (m) while keeping the cycle time constant. Conversely, if the goal is to minimize the cycle time (c) with a predetermined number of stations, it is termed a Type-II problem which is used for rebalancing and optimization of already installed stations. Type-E problems, also known as ALBP-E, emerge when both objectives are combined, aiming to simultaneously minimize both cycle time and the number of stations. Type-F problems, on the other hand, focus on finding a feasible balance for given values of cycle time and number of stations (Jonnalagedda & Dabade, 2014; Aufy & Kassam, 2020).

The mathematical or heuristic algorithms used in assembly line balancing deal with the optimization of the primary objective in these two general types of balancing problems. Urban (1998), first evaluated the problems in the literature as Straight Assembly Lines, and then evaluated them as U-Shape with the "Phantom Network Diagram". He proposed and obtained solutions with Integer Programming. He also calculated the Maximum Ranked Positional Weight and compared the same line over three solutions U-Line Integer Programming, U-Line MRPW, and Straight Line Integer Programming. The calculations made in the proposed method are applied to well-known assembly line templates in the literature such as Killbridge, Sawyer, and Heskiaoff.

There are numerous studies in the literature dealing with various types of assembly line balancing problems considering different objectives (Sivasankaran & Shahabudeen, 2014).

However, the scope of this study is limited to providing smoothness after solving the Type-I assembly line balancing problem via integer programming. Smothness which provides the fair distribution of idle times among stations is important with respect to ergonomics in general and specifically about the motivation of the workers. Hence, the effect of uneven workload can be mitigated via providing smoothness. Workload smoothing line balancing problems are not possible to solve exactly using mixed-integer linear programming (MILP), and heuristic approaches take place in such cases for complex problems (Dinler & Tural, 2021). Different from the existing studies, the smoothness is provided via two approaches. One of them is the classical one which basically considers the minimization of the sum of square of workload deviations and the other proposed one focuses on the minimization of the maximum idle times among the stations. For this aim, firstly, the integer programming of straight assembly line for an existing problem in literature (Sawyer problem) is provided for Type-I problem, modeled and solved via Gurobi optimization program. Afterward, the smoothness is provided via two approaches and comparative analyses are given.

The rest of the paper is organized as follows; the second section includes the mathematical modeling for straight assembly line problems. The application of the proposed methodology for Sawyer problem is given in the third section. Finally, the conclusions are provided with the references following.

2. Mathematical Modelling for Straight Assembly Line

The mathematical modeling for the straight assembly line problem for the minimization of stations is provided as follows:

$$Minimize \ \sum_{j=\lceil m_{min}\rceil+1}^{m_{max}} z_j \tag{1}$$

$$subject\ to: \sum_{j=1}^{m_{max}} x_{ij} = 1 \ \ for\ i = 1, \dots, n, \tag{2}$$

$$\sum_{i=1}^{n} t_i \left(x_{ij}\right) \leq C \ \ for\ j = 1, \dots, \lceil m_{min}\rceil, \tag{3}$$

$$\sum_{i=1}^{n} t_i \left(x_{ij}\right) \leq C z_j \;\; for\; j = \lceil m_{min} \rceil + 1, \dots, m_{max}, \tag{4}$$

$$\sum_{j=1}^{m_{max}} (m_{max} - j + 1) \left(x_{rj} - x_{sj}\right) \geq 0 \;\; for\; all\; (r, s) \;\in P, \tag{5}$$

$$x_{ij}, z_{ij} \in \{0, 1\}\, for\; all\; i, j. \tag{6}$$

The objective function given in Equation (1) considers the minimization of stations. Since there will be at least the number of minimum required stations, the lower limit of summation is one plus it. Constraint (2) ensures that each work element is assigned to exactly one station. Constraints (3) and (4) ensure that the station time does not exceed the pre-defined cycle time. The precedence matrix P is defined according to the precedence relations of each task as *P = (r,s).* Finally, constraint (5) enforces the precedence relations among tasks and the types of decision variables are given in constraint (6).

After solving the problem and finding the optimum number of stations, in the second stage, the model is solved for the minimization of the smoothness index. As stated before, two approaches are used here; the classical one and the proposed min-max approach.

The classical approach for the smoothness index

When calculating the smoothness index, the square root of the sum of the squares of the differences between the maximum work time and the duration of each station is used (Ponnambalam et al., 1999). The related formulation for the classical smoothness is provided in Equation (7). “SI” indicates the smoothness index, “ST_{max}” stands for the maximum of station times, and “ST_i” stands for the time of station i.

$$SI = \sqrt{\textstyle\sum_{t=1}^{m} (ST_{max} - ST_i)^2} \tag{7}$$

When the objective function provided in Equation (7) is used, the model becomes non-linear and the optimum solution is not guaranteed and it takes a long time to obtain a good feasible solution.

The proposed min-max approach for the smoothness index

In the proposed smoothness index calculation, the focus point is to minimize the maximum of idle times. Hence the objective function and the constraints are modified and modelled in Gurobi optimization program. The objective function is given in Equation (8) and the added constraint is provided in Equation (9). The main difference between the proposed approach and the classical one is that there doesn't exist nonlinearity in the proposed approach. Hence, the computational advantage is provided.

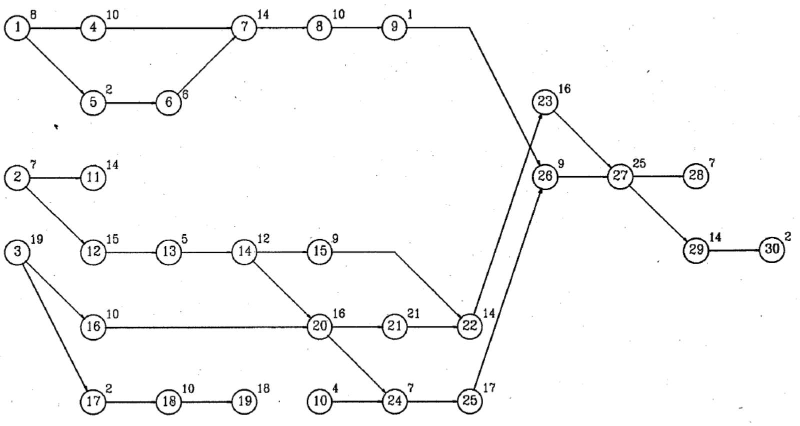
$$Minimize\ \lambda \tag{8}$$

$$C - \sum_{i=1}^{n} t_i \left(x_{ij}\right) \leq \ \lambda\, for\ all\ j \tag{9}$$

3. Application of the Proposed Methodology for Sawyer Problem

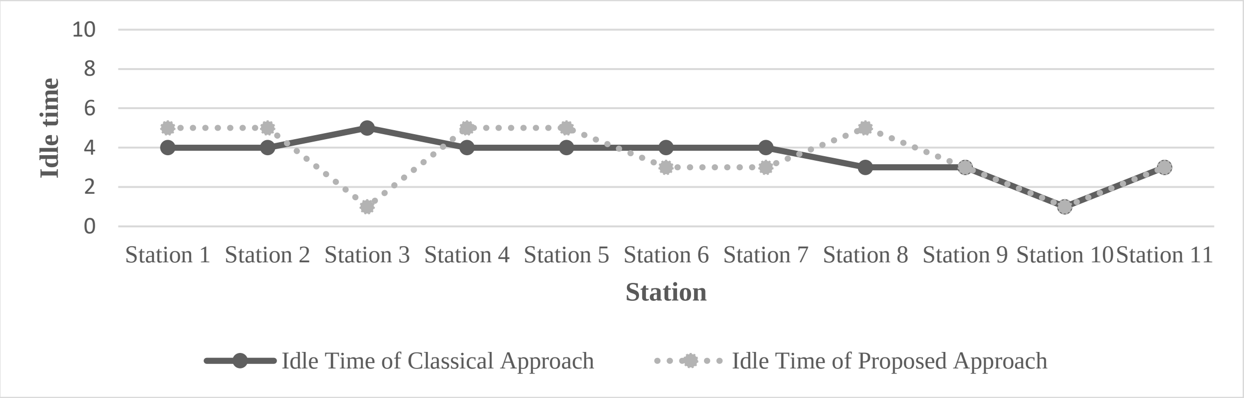
The classical Sawyer problem in the literature is considered for the application of the proposed methodology. The problem is in the type of straight assembly line and the work element times are deterministic. Each task can be done at any station without station restrictions. There are 30 work elements in Sawyer's problem and their precedence relations can be seen in Figure 3.1. The problem is solved with integer programming to achieve the minimum number of stations initially. The dataset for the example problem is gathered from URL-1.

Figure 3.1: 30 Task Sawyer Problem Precedence Diagram and Task Times



The problem is solved with constraints and equations provided in the previous section with different cycle times. For the first calculation, cycle time is considered as 33, and smoothness index is found as 20.616 initially, 9.055 with classical method, and 9.798 with the proposed min-max approach, while the number of stations are same. When we take a closer look into that specific calculation, the distribution of idle times among the stations with two different approaches is provided in Figure 3.2.

Figure 3.2: *Idle Times of each station for 33s cycle time with two different approaches*



It can be seen clearly that each station's idle times vary and make big gaps with the average station idle time when the problem is solved without considering the smoothness of the assembly line. Similar results were found with two different approaches when the smoothness is considered.

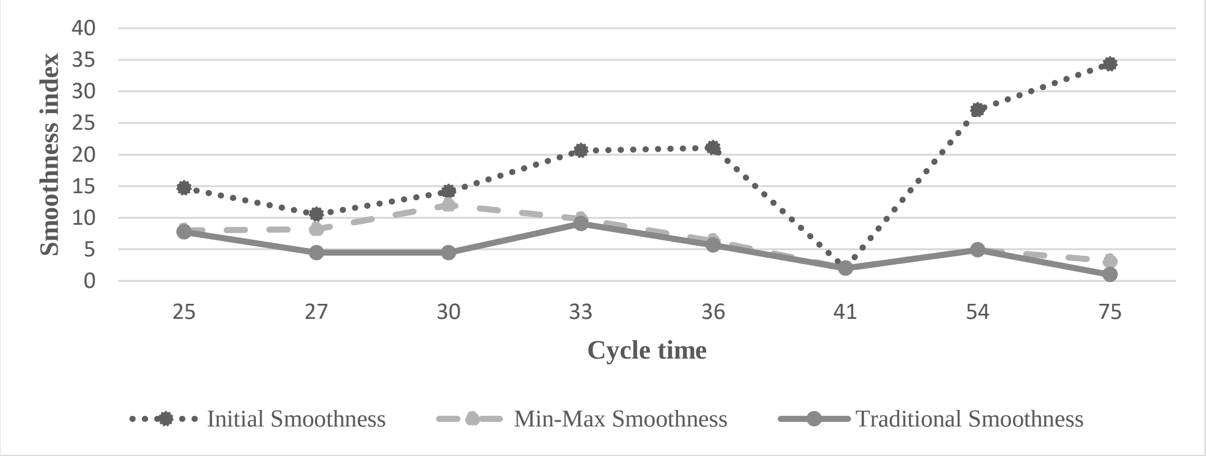
The solution is extended with more cycle time options, initial results without smoothing, classical method of smoothness calculation, and proposed methodology results are given in Table 3.1, regarding smoothness indexes and maximum idle time among the stations.

Table 3.1 *Comparison of Solution Methods According to SI and Maximum Idle Time*

Cycle time	**Initial solution**		**The classical method**		**The proposed method (min-max)**	
	Maximum Idle Time	**Smoothness Index**	**Maximum Idle Time**	**Smoothness Index**	**Maximum Idle Time**	**Smoothness Index**
25	14	14.697	3	7.746	3	8.000
27	7	10.535	3	4.472	3	8.185
30	9	14.142	5	4.472	5	12.000
33	15	20.616	5	9.055	5	9.798
36	20	21.071	5	5.657	5	6.325
41	1	2.000	1	2.000	1	2.000
54	20	27.092	8	4.899	8	4.899
75	34	34.531	11	1.000	11	3.162

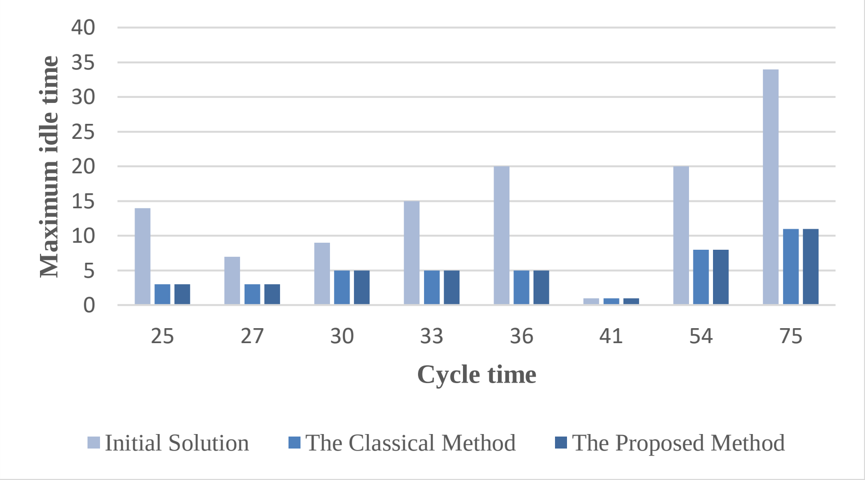
The proposed approach found the same results for maximum idle time among the stations for all of them, meanwhile, smoothness indexes are approximate to the actual smoothness index solution, as can be seen in Figure 3.3. However, depending on the linear structure of the proposed approach, it is easier to obtain approximate results in larger problems.

Figure 3.3: *Smoothness Index Comparison of Results*



On the other hand, it is also possible to use the proposed approach instead of the classical method when the maximum idle time is considered in smoothness minimization. The proposed approach provides the minimization of maximum idle time as seen in Figure 3.4 and can be regarded as an alternative method.

Figure 3.4: *Maximum Idle Times Comparison of Results*



3. Conclusions

The results showed that the proposed min-max smoothness approach gives approximate and acceptable results in many cases when compared to the non-linear smoothness minimization approach. Different from the classic smoothness calculations, minimization of maximum idle time also creates a fair work environment and increases collaboration among team members. The presented method can be helpful in reducing the calculation time of more complicated problems. In addition, since the function is linear, finding the optimal solution in complex problems is more assured. Hence, it can be concluded that the proposed approach is a good alternative for the classic smoothness calculations.

As future work, larger problems can be solved to compare calculation times and solution approximations. Other layout types rather than straight lines such as u-shaped assembly lines might be considered to measure the effectiveness of the provided approach. Last but not the least, multi and mixed model assembly lines can be considered for the application of the proposed approach.

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URL-1 https://assembly-line-balancing.de Access date: 19.04.2024