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SLIP EFFECTS ON MHD STAGNATION-POINT FLOW OF CARREAU FLUID PAST A PERMEABLE SHRINKING SHEET

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Abstract

Carreau fluid is a type of generalized Newtonian fluid where viscosity depends upon the shear rate of the fluid and then uses it to obtain a formulation for the boundary layer equations of the Carreau fluid. The objective of the present study is to analyze the development of the slip effect on the MHD stagnation-point flow of Carreau fluid past a shrinking sheet. The mathematical modeling of Carreau fluid has been developed for boundary layer problem and the governing partial differential equations are transformed into ordinary differential equation using self-similarity transformation. The effect of velocity slip is taken into account and controlled by non-dimensional parameter. The *dual solutions are obtained when the sheet is shrunk. The study shows that the skin friction decreases with an increase in velocity slip.*

Keywords

Carreau Fluid, Boundary Layer, Shrinking Sheet, Slip Condition

1. Introduction

The study on the heat and mass transfer characteristics about convection of non-Newtonian fluids is of much importance because of practical engineering applications, such as catalytic reactors (Cohen & Maron, 1983), the filtration devices (Holeschovsky & Cooney, 1991) and blood plasmaphosresis devices (Beaudoin & Jaffrin, 1989). The convective heat transfer mechanisms of non-Newtonian fluids are the subject of considerable works and are well understood today. The Carreau viscosity model is one of the non-Newtonian fluid model in which constitutive relationship is valid for low and high shear rates. The peristaltic transport of Carreau fluid in an asymmetric channel has been analyzed by Ali & Hayat (2007). The flow of Carreus fluid down an incline free surface was examined by Tshehla (2011). In another article, Olajuwon (2011) studied convective heat mass transfer in a hydromagnetic flow Carreau fluid past a vertical porous plat in presence of thermal radiation and thermal diffusion. Later, Hayat et al. (2014) discussed boundary layer flow of Carreau fluid over a convectively heated stretching sheet. Akhbar et al. (2014) analyzed MHD stagnation point flow of Carreau fluid toward a permeable shrinking sheet and they obtained the dual solution. Naganthran & Nazar (2016) extended Akhbar et al. (2014) paper to analyze the stability of dual solution and they showed that the first solution is stable and the second solution is unstable. Very recently, Azam et al. (2017) investigated the unsteady magnetohydrodynamic (MHD) axisymmetric flow of Carreau nanofluid over a radially stretching sheet. It should be mentioned that the present work is to extend paper by Akhbar et al. (2014) on MHD stagnation point flow of Carreau fluid toward a permeable shrinking sheet with slip condition.

2. Problem Formulation

Consider the steady two-dimensional flow of a Carreau fluid near the stagnation-point on a vertical flat plate of uniform ambient temperature T_∞. It is assumed that the velocity distribution far from the surface (potential flow) is given by $u_e(x) = a\,x$, where $a < 0$ is a shrinking rate. The constitutive equation for non-newtonian Carreau fluid can be written as,

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + \left(\Gamma \bar{\dot{\gamma}}\right)^2\right]^{(n-1/2)} \quad (1)$$

where η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant and n si the dimensionless power-law index. If n = 1, then $\eta = \eta_0$, that is the Newtonian viscosity of the fluid.

The following equations defines $\bar{\dot{\gamma}}$ as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{\gamma}}_{ij}\, \bar{\dot{\gamma}}_{ij}} = \sqrt{\frac{1}{2}\prod_{\bar{\dot{\gamma}}}} \quad (2)$$

where $\prod_{\bar{\dot{\gamma}}}$ is the strain rate tensor. Extra stress tensor for Carreau fluid is

$$\bar{\tau}_{ij} = -\eta_0 \left[1 + \left(\frac{n-1}{2}\right)\left(\Gamma \bar{\dot{\gamma}}\right)^2\right] \bar{\dot{\gamma}}_{ij} \quad (3)$$

where $\bar{\tau}_{ij}, i, j = 1,2,3$ are the components of the extra stress tensor.

Under these assumption, the boundary layer equations which govern this problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{d u_e}{d x} + \nu\frac{\partial^2 u}{\partial y^2} + \nu\frac{3(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(u_e - u) \quad (5)$$

subject to the boundary conditions n = 1

$$\begin{aligned} &v = v_w(x), \quad u = ax + L\left(\frac{\partial u}{\partial y}\right), \qquad \text{at} \quad y = 0 \\ &u = u_e(x) = b\,x, \quad \text{as} \quad y \to \infty \end{aligned} \quad (6)$$

where b is a positive constant. We introduce now the following similarity variables

$$\psi = \sqrt{b\nu}\, x f(\eta), \qquad \eta = \sqrt{b/\nu}\, y \quad (7)$$

where ψ is the stream function which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Substituting (7) into Eqs. (4) and (5), the following set of ordinary differential equations results in

$$f''' - (f')^2 + f\,f'' + \frac{3(n-1)We^2}{2} f'''(f'')^2 + M^2(1 - f'^2) = 0 \quad (8)$$

and the boundary conditions (6) become

$$f(0) = s, \quad f'(0) = \lambda + \chi f''(0)$$
$$f'(\eta) \to 1 \quad \text{as} \quad \eta \to \infty \quad (9)$$

where primes denote differentiation with respect to η, $s = -v_w/(av)^{1/2}$ is the Prandtl number, and λ is the stretching/shrinking parameter, We is the Weissenberg number, M is the magnetic parameter and χ is a velocity slip parameter.

The physical quantities of interest are the skin friction coefficient C_f which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2(x)}, \quad (10)$$

where τ_w is the skin friction or shear stress along the stretching surface which is given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (11)$$

Using (7), we get

$$\mathrm{Re}_x^{1/2} C_f = f''(0) \quad (12)$$

where $\mathrm{Re}_x^{1/2} C_f$ is the local Reynolds number.

3. Results and Discussion

The nonlinear ordinary differential equation (8) subject to the boundary conditions (9) were solved numerically using the shooting method. This well-known technique is an iterative algorithm which attempts to identify appropriate initial conditions for a related initial value problem (IVP) that provides the solution to the original boundary value problem (BVP). The shooting method is based on MAPLE "dsolve" command and MAPLE implementation, "shoot" (Meade et al. 1996).

The results are given to carry out a parametric study showing the influences of the non-dimensional parameters, namely the shrinking parameter λ , suction parameter, s and the velocity slip parameter, χ.

In order to validate the accuracy of the numerical method used, the present results for the skin friction coefficient $f''(0)$ for various values of λ are compared with those of Naganthran and Nazar (2016) and Akbar et al. (2014), as shown in Table 1. The results are in good agreement, thus lending confidence to the accuracy of the present results. Variation of the skin friction coefficient $f''(0)$ with χ are presented in Table 2. The results indicate that the increase of velocity slip parameter decrease the skin friction coefficient $f''(0)$ for first solution. As noted by Naganthran and Nazar (2016), the first solution is stable and physically realizable, while the second solution is unstable. In this study, our primary focus on the effects of the shrinking parameter, slip coefficients and suction parameter.

The influence of the parameters χ and s on the velocity profiles clearly shown Figures 1 and 2. The velocity of the fluids increases with slip parameter χ decrease. This is because slip parameter can generate the vorticity of shrinking velocity at its enhance the velocity the surface. We also observed that with an increase in slip parameters, the boundary layer thickness decreases. Figure 2 shows that $f'(\eta)$ increase with s because suction reduces drag force in order to avoid boundary layer separation. It can be seen that dual solution are also occur in both Figures 1 and 2, and the boundary layer thicknesses are larger for second solution than for the first solution. Figures 1 and 2 also show that they satisfy the far field boundary conditions (9) asymptotically, which support the numerical results obtained.

Table 1: *Comparison of the values of skin friction coefficient with M = 0, We = 0, s = 0 and n = 1.*

λ	Present results	Naganthran and Nazar [9]	Akbar et al. [8]
-1.2465	0.5843	0.5844	0.5543
	(0.5543)	(0.5542)	(0.5542)
-1.20	0.9325	0.9325	0.9325
	(0.2336)	(0.2336)	(0.2336)
-1.15	1.0822	1.0822	1.0822
	(0.1167)	(0.1167)	(0.1167)

-1.10	1.1867	1.1867	1.1867
	(0.0492)	(0.0492)	(0.0492)
-1.00	1.3288	1.3288	1.3288
	(0)	(0)	(0)
-0.75	1.4893	1.4893	1.4893
-0.50	1.4957	1.4956	1.4956
	(-6.9731)	-	-
-0.25	1.4022	1.4022	1.4022
0	1.2326	1.2326	1.2326
0.10	1.1466	1.1466	1.1466
0.20	1.0511	1.0511	1.0511
0.50	0.7133	0.7133	0.7133
1.00	0	0	0
2.00	-1.8873	-1.8873	-1.8873
5.00	-10.2647	-10.2647	-10.2647
	(-10.7719)	-	-

*( ) SECOND SOLUTION

Table 2: *Values of skin friction coefficient with M = 0.5, We = 0.3, s = 5,* $\lambda = -2$ *and n = 2 for different values of velocity slip parameter* χ.

χ		First Solution	Second Solution
0		6.5132	-5.4351
0.1		5.9736	-4.95001
0.2		5.4050	-4.4993
0.3		4.8551	-4.0816
0.4		4.3525	-3.7004

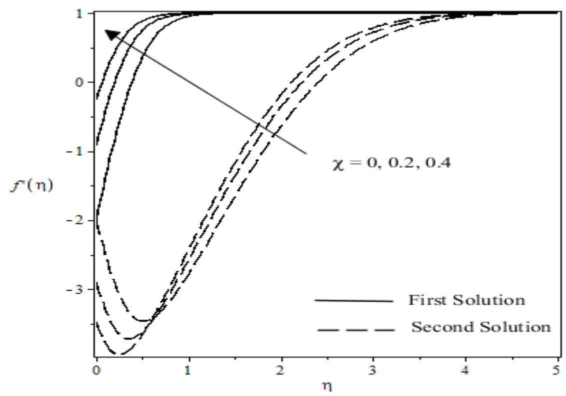

Figure 1: Effects of χ on the velocity profile $f'(\eta)$ when $M = 0.5$, $We = 0.3$, $s = 5$, $\lambda = -2$ *and* $n = 2$

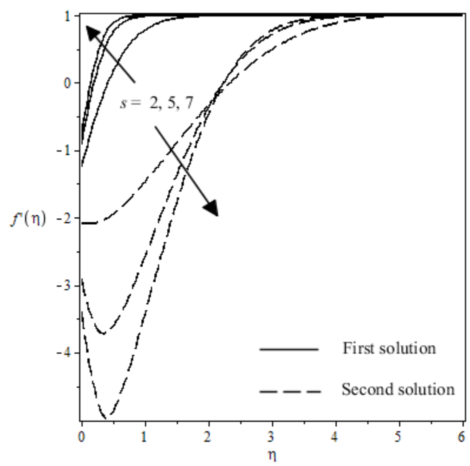

Figure 2: Effects of s on the velocity profile $f'(\eta)$ when $M = 0.5$, $We = 0.3$, $\chi = 0.2$, $\lambda = -2$ *and* $n = 2$

4. Conclusions

The present study explores the momentum boundary layers due to the motion of non-Newtonian Carreau fluid with slip condition. The equations for Newtonian case can be recovered from the derived equations as limiting cases. The shooting method was used to solve the developed mathematical formulations and the calculated results are compared with the existing literature in the limiting case (no slip condition). It is noted that the skin friction increase when slip parameter decrease for first solutions. The effects of the governing parameters on velocity profiles are presented graphically. It is observed that the suction parameter decrease the velocity field while increases the boundary layer thickness. Since the problems have dual solutions, the stability analysis will be carried out to identify which solution is stable and physically realizable for future work.

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