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OPTIMAL CONTROL OF NEUTRAL LIPIDS IN MICROALGAE PRODUCTION WITH NUTRIENT LIMITATION

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Abstract

Consumer demand for fuel was increasing, while the supply of fuel has began dwindling. Therefore, it is necessary to undertake an effort to develop a renewable alternative energy such as usage of microalgae. Microalgae have four main components of substance i.e carbohydrates, proteins, nucleic acid and lipids. The relatively high lipid levels can be used as a source of biomass with using light, glucose, nutrients, carbon dioxide and water. Nutrient concentration is modified to keep the concentration of biomass through the dilution rate. In *addition, carbon dioxide regulated also influence of microalage production in photobioreactor. Thereby, we used dilution rate and carbon dioxide mobilization as optimal control using Pontryagin Maximum Principle method to increased biomass and quota lipid production. Hence, the result is biomass increased as 4,5678% and quota lipid increased 44, 9727%.*

Keywords

Microalgae, Carbon Dioxide, Nutrient Limitation, Optimal Control, Pontryagin Maximum Principle.

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1. Introduction

Indonesian citizen always increased in every year, these phenomena cause human needed increased too, especially fuel consume. Mostly, the fuel will depleting condition, thus natural resources changed into biofuel using microalgae. It's have highest potential as biomass renewable with contains many lipids. As a photosynthetic organisms, microalgae also contains chlorophyll to produced biomass through photosynthetic cycle using carbon dioxide and light. Microalgae contains lipid up to 70%, they can produced biomass to much between the other plants. Microalgae is microscopic plant who required some elements such as light, glucose, nutrient, carbon dioxide and water to used their growth up process. Microalgae can absorb many carbon dioxide to reduced global warming effect, these also to use as carbon sources. They have changed potential to absorb heavy metals and waste water, thus can reduced the pollution. Algae has a same photosynthetic cycle such as the other plant, if in the day they did photosynthetic process to produced oxygen O_2, but in the night they did respiration process to produced carbon dioxide CO_2.

Many people did research to develop microalgae growth to produced biomass effectively. In the previous research, algae a diversified *Isochrysis aff. Galbana* explored about optimal strategy using mathematic model neutral lipid production in microalgae growth with nitrogen limitation (Mairet etc, 2011). The other microalgae research in a few years ago about optimal control to increased alga growth and produced biomass was explored and made by (Hajar and Mardlijah, 2015), (Nasria and Mardlijah, 2016), (Nailul & Mardlijah, 2016) and (Jamil, Mardlijah, Suharmadi, and Lukman, 2017). To optimize microalgae production with dilution rate of nutrient as variable optimal control have been solve by (Nasria and Mardlijah, 2016), (Hajar and Mardlijah, 2015) used carbon dioxide as parameter control to optimize the algae growth and (Nailul and Mardlijah, 2016) was already made research about optimal control in feeding treatment to shown the microalgae growth. (Jamil, Mardlijah, Suharmadi, Lukman, 2017) was develop optimal control used carbon dioxide and nutrition flow as variable control to shown algae growth.

Hence, with gained modify mathematics model from (Nasria and Mardlijah, 2016) and combine both to controlling dilution rate nutrient and carbon dioxide to optimize biomass and lipid quota using Pontryagin Maximum Principle method with objective function to solve the problem about optimal control. The paper has been written as follows : The modify mathematics model will be written in Section 2. The stability system will be written in Section 3. Pontryagin Maximum Principle will be written in Section 4. The optimal control using derivation will be written in Section 5. The numerical analysis and result will be written in the Section 6. In the last section will be written conclusion about optimal control to optimize microalgae production.

2. The Mathematics Model of Microalgae Production

We followed (Mairet etc, 2011) research result such as the mathematics model of microalgae production, variabel, parameters and equations to used develop these research. In this research added one parameter, it is δ or carbon dioxide accommodation to mobilization. To proofed δ as influence in the mathematics model as follows this describes: "Photosynthetic and respiration cycle of microalage are rotate daily. Photosynthetic produced oxygen and carbohydrate, later respiration produced carbon dioxide. Respiration is the sum of a basal respiration proportional to biomass and term proportional to cell activity, and thus to the growth rate (Bernard, 2010). Thereby, respiration process can made cell mortality, in the other side it's can produced carbon dioxide. Thus, CO_2 is released, can catch up accommodated these while respiration process in the night. The carbon dioxide still used into the photosynthetic process."

The **Table 1** described about non linear equation from the microalgae production with variables and parameters. The dynamic system from microalgae production to optimize the biomass and quota lipid retrieved from derivation equation such as follow:

$$\dot{s} = Ds_{in} - \rho_m \frac{s}{s+K_s} x - D_s \quad (1)$$

$$\dot{q_n} = \rho_m \frac{s}{s+K_s} - \bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) q_n \quad (2)$$

$$\dot{x} = \bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) x - D_x + \delta \rho_m \frac{s}{s+K_s} x \quad (3)$$

$$\dot{q_l} = (\beta q_n - q_l)\bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) - \gamma \rho_m \frac{s}{s+K_s} \quad (4)$$

$$\dot{q_f} = -q_f\bar{\mu}\left(1-\frac{Q_0}{q_n}\right)+(\alpha+\beta-\delta)\rho_m\frac{s}{s+K_s} \quad (5)$$

thus we can described the initial condition where for $s(0)=s_0, q_n(0)=q_{n0}, x(0)=x_0$, $q_l(0)=q_{l0}$ and $q_f(0)=q_{f0}$.

Table 1: *Variables and Parameters (Mairet, etc, 2011)*

Notation	Description
s	Nutrient Substrate
s_{in}	Concentration of nutrients are included
q_n	Nitrogen quota
x	Biomass
q_l	Lipid quota
q_f	Carbon quota
D	Dilution rate
$\bar{\mu}$	Maximum groth rate
ρ	Absorption rate
β	Fatty acid synthesis coefficient
γ	Fatty acid mobiliation coefficient
α	Protein synthesis coefficient
δ	Carbon dioxide accommodation to mobilization
Q_0	Minimum nitrogen quota
K_s	Half-saturation constant

3. Stability System

In this section will be shown the stability system. Using the mathematics model in the previous section, to solve with SageMath (Benjamin, 2016) thus we getting the point of stability such as below,

$$\lambda_1 = -D$$

$$\lambda_2 = \mu$$

$$\lambda_3 = \mu\left(1-\frac{(K_s+s_{in})Q_0\mu}{K_sQ_0\mu+Q_0\mu s_{in}+\rho_m s_{in}}\right)-D+\frac{\delta\rho_m s_{in}}{K_s+s_{in}}$$

$$\lambda_4 = -\mu\left(1-\frac{(K_s+s_{in})Q_0\mu}{K_sQ_0\mu+Q_0\mu s_{in}+\rho_m s_{in}}\right)$$

$$\lambda_5 = -\mu\left(1-\frac{(K_s+s_{in})Q_0\mu}{K_sQ_0\mu+Q_0\mu s_{in}+\rho_m s_{in}}\right)$$

where λ_2 and λ_3 have boundary condition between $[-1,0\} \cup \{0,1]$. The lower bound is -1 its mean maximum growth rate decreased. The upper bound is 1 its mean maximum growth rate increased.

4. Optimal Control

In this section will be described about optimal control used dilution rate of nutrient and carbon dioxide accommodation to maximize biomass and lipid quota production from microalgae. Thereby, to solve the problem we established an objective function such as follow,

$$J(u_1, u_2) = \int_{t_0}^{t_f} \left(x(t) + q_l(t) - \frac{1}{2} c_1 u_1(t)^2 - \frac{1}{2} c_2 u_2(t)^2 \right) dt \quad (6)$$

for J as objective function, u_1 as control variable of dilution rate and u_2 as control variable of carbon dioxide. Therefore, c_1 described weight factor in the dilution rate of nutrient and c_2 described weight factor in the carbon dioxide accommodation to mobilization flow. These integral equations began from t_0 as start value of time and t_f as final value of time.

5. Pontryagin Maximum Principle

In this section described Pontryagin Maximum Principle is used for optimal control to find the best solution control from dynamic system in the microalgae production. These method used the Hamiltonian function with added λ as parameters control. To maximize the microalgae production we used the Hamiltonian function with to solved derived equation from the state function, costate function and boundary conditions.

Using the Hamiltonian function, we can times the objective function equation 6 with the microalgae production model 1 to 5, hence we get Hamiltonian function such as follow,

$$\begin{aligned} H = \left(s, q_n, x, q_l, q_f, u_1, u_2, \lambda\right) &= x(t) + q_l(t) - \frac{1}{2} c_1 u_1(t)^2 - \frac{1}{2} c_2 u_2(t)^2 + \sum_{i=1}^{5} \lambda_i f_i \\ &= x(t) + q_l(t) - \frac{1}{2} c_1 u_1(t)^2 - \frac{1}{2} c_2 u_2(t)^2 + \\ &\lambda_1 \left[D s_{in} - \rho_m \frac{s}{s+K_s} x - D_s \right] + \\ &\lambda_2 \left[\rho_m \frac{s}{s+K_s} - \bar{\mu} \left(1 - \frac{Q_0}{q_n}\right) q_n \right] + \\ &\lambda_3 \left[\bar{\mu} \left(1 - \frac{Q_0}{q_n}\right) x - D_x + \delta \rho_m \frac{s}{s+K_s} x \right] + \end{aligned}$$

$$\lambda_4\left[(\beta q_n - q_l)\bar{\mu}\left(1-\frac{Q_0}{q_n}\right) - \gamma\rho_m\frac{s}{s+K_s}\right] +$$

$$\lambda_5\left[-q_f\bar{\mu}\left(1-\frac{Q_0}{q_n}\right) + (\alpha+\beta-\delta)\rho_m\frac{s}{s+K_s}\right] +$$

The state function result of derivation from Hamiltonian function against λ_1 until λ_5 such as follow:

$$\frac{\partial H}{\partial \lambda_1} = Ds_{in} - \rho_m\frac{s}{s+K_s}x - D_s$$

$$\frac{\partial H}{\partial \lambda_2} = \rho_m\frac{s}{s+K_s} - \bar{\mu}\left(1-\frac{Q_0}{q_n}\right)q_n$$

$$\frac{\partial H}{\partial \lambda_3} = \bar{\mu}\left(1-\frac{Q_0}{q_n}\right)x - D_x + \delta\rho_m\frac{s}{s+K_s}x$$

$$\frac{\partial H}{\partial \lambda_4} = (\beta q_n - q_l)\bar{\mu}\left(1-\frac{Q_0}{q_n}\right) - \gamma\rho_m\frac{s}{s+K_s}$$

$$\frac{\partial H}{\partial \lambda_5} = -q_f\bar{\mu}\left(1-\frac{Q_0}{q_n}\right) + (\alpha+\beta-\delta)\rho_m\frac{s}{s+K_s}$$

and the costate function result of derivation from Hamiltonian function against variable s, q_n, x, q_l, q_f such as follow :

$$\frac{\partial \lambda_1}{dt} = \frac{\partial H}{\partial s}$$

$$= \frac{\lambda_3\rho_m u_2 x}{K_s+s} - \frac{\lambda_3\rho_m u_2 x}{(K_s+s)^2} - \frac{\gamma\lambda_4\rho_m}{K_s+s} + \frac{\gamma\lambda_4\rho_m}{(K_s+s)^2} + \frac{(\alpha+\beta-u_2)\lambda_5\rho_m}{K_s+s} - \frac{(\alpha+\beta-u_2)\lambda_5\rho_m}{(K_s+s)^2}$$

$$-\lambda_1 u_1 - \frac{\lambda_1\rho_m x}{K_s+s} + \frac{\lambda_1\rho_m x}{(K_s+s)^2} + \frac{\lambda_2\rho_m}{K_s+s} + \frac{\lambda_2\rho_m}{(K_s+s)^2}$$

$$\frac{\partial \lambda_2}{dt} = \frac{\partial H}{\partial q_n}$$

$$= -\beta\lambda_4\mu\left(\frac{Q_0}{q_n}-1\right) + \lambda_2\mu\left(\frac{Q_0}{q_n}-1\right) + \frac{(\beta q_n - q_l)Q_0\lambda_4\mu}{(q_n)^2} - \frac{Q_0\lambda_5\mu q_f}{(q_n)^2} - \frac{Q_0\lambda_2\mu}{q_n} + \frac{Q_0\lambda_3\mu x}{(q_n)^2}$$

$$\frac{\partial \lambda_3}{dt} = \frac{\partial H}{\partial x}$$

$$= \frac{\lambda_3\rho_m s u_2}{K_s+s} - \lambda_3\mu\left(\frac{Q_0}{q_n}-1\right) - \frac{\lambda_1\rho_m s}{K_s+s} - \lambda_3 u_1 + 1$$

$$\frac{\partial \lambda_4}{dt} = \frac{\partial H}{\partial q_l}$$

$$= \lambda_4 \mu \left(\frac{Q_0}{q_n} - 1\right) + 1$$

$$\frac{\partial \lambda_5}{dt} = \frac{\partial H}{\partial q_f}$$

$$= \lambda_5 \mu \left(\frac{Q_0}{q_n} - 1\right) + 1$$

The derivation of Hamiltonian function against control parameters u_1 and u_2 are given such as follow :

$$\frac{\partial H}{\partial u_1(t)} = 0$$

$$u_1(t) = -\frac{\lambda_1(s - s_{in}) + \lambda_3 x}{c_1}$$

If $0 \leq u_1(t) \leq 1$, then

$$u_1(t) = \begin{cases} 0 & \overline{u_1(t)} \leq 0 \\ \overline{u_1(t)}, & 0 \leq \overline{u_1(t)} < 1 \\ 1 & \overline{u_1(t)} \geq 1 \end{cases}$$

Thus, we can written the optimal control u_1 such as follow :

$$u_1(t)^* = min\left(1, max\left(0, \overline{u_1(t)}\right)\right)$$

Or another equation such as

$$u_1(t)^* = min\left(1, max\left(0, -\frac{\lambda_1(s - s_{in}) + \lambda_3 x}{c_1}\right)\right)$$

$$\frac{\partial H}{\partial u_2(t)} = 0$$

$$u_2(t) = \frac{\frac{\lambda_3 \rho_m s x}{K_s + s} - \frac{\lambda_5 \rho_m s}{K_s + s}}{c_2}$$

If $0 \leq u_2(t) \leq 1$, then

$$u_2(t) = \begin{cases} 0 & \overline{u_2(t)} \leq 0 \\ \overline{u_2(t)}, & 0 \leq \overline{u_2(t)} < 1 \\ 1 & \overline{u_2(t)} \geq 1 \end{cases}$$

Thus, we can written the optimal control u_1 such as follow:

$$u_2(t)^* = min\left(1, max\left(0, \overline{u_2(t)}\right)\right)$$

Or another equation such as

$$u_2(t)^* = min\left(1, max\left(0, \frac{\frac{\lambda_3\rho_m sx}{K_s+s} - \frac{\lambda_5\rho_m s}{K_s+s}}{c_2}\right)\right)$$

To maximize the result of the optimal system, we can substituted the parameter control $u_1(t)^*$ and $u_2(t)^*$ into *state* equations and *costate* equation. Therefore, we can written the substitution such as follow :

$$\frac{\partial H}{\partial \lambda_1} = u_1(t)^* s_{in} - \rho_m \frac{s}{s+K_s} x - u_1(t)^*$$

$$\frac{\partial H}{\partial \lambda_2} = \rho_m \frac{s}{s+K_s} - \bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) q_n$$

$$\frac{\partial H}{\partial \lambda_3} = \bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) x - u_1(t)^* + u_2(t)^* \rho_m \frac{s}{s+K_s} x$$

$$\frac{\partial H}{\partial \lambda_4} = (\beta q_n - q_l)\bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) - \gamma\rho_m \frac{s}{s+K_s}$$

$$\frac{\partial H}{\partial \lambda_5} = -q_f\bar{\mu}\left(1 - \frac{Q_0}{q_n}\right) + (\alpha + \beta - u_2(t)^*)\rho_m \frac{s}{s+K_s}$$

$$\frac{\partial \lambda_1}{dt} = \frac{\partial H}{\partial s}$$

$$= \frac{\lambda_3\rho_m u_2(t)^* x}{K_s+s} - \frac{\lambda_3\rho_m u_2(t)^* x}{(K_s+s)^2} - \frac{\gamma\lambda_4\rho_m}{K_s+s} + \frac{\gamma\lambda_4\rho_m}{(K_s+s)^2} + \frac{(\alpha+\beta-u_2(t)^*)\lambda_5\rho_m}{K_s+s} - \frac{(\alpha+\beta-u_2(t)^*)\lambda_5\rho_m}{(K_s+s)^2} - \lambda_1 u_1(t)^* - \frac{\lambda_1\rho_m x}{K_s+s} + \frac{\lambda_1\rho_m x}{(K_s+s)^2} + \frac{\lambda_2\rho_m}{K_s+s} + \frac{\lambda_2\rho_m}{(K_s+s)^2}$$

$$\frac{\partial \lambda_2}{dt} = \frac{\partial H}{\partial q_n}$$

$$= -\beta\lambda_4\mu\left(\frac{Q_0}{q_n} - 1\right) + \lambda_2\mu\left(\frac{Q_0}{q_n} - 1\right) + \frac{(\beta q_n - q_l)Q_0\lambda_4\mu}{(q_n)^2} - \frac{Q_0\lambda_5\mu q_f}{(q_n)^2} - \frac{Q_0\lambda_2\mu}{q_n} + \frac{Q_0\lambda_3\mu x}{(q_n)^2}$$

$$\frac{\partial \lambda_3}{dt} = \frac{\partial H}{\partial x}$$

$$= \frac{\lambda_3\rho_m s u_2(t)^*}{K_s+s} - \lambda_3\mu\left(\frac{Q_0}{q_n} - 1\right) - \frac{\lambda_1\rho_m s}{K_s+s} - \lambda_3 u_1(t)^* + 1$$

$$\frac{\partial \lambda_4}{dt} = \frac{\partial H}{\partial q_l}$$

$$= \lambda_4\mu\left(\frac{Q_0}{q_n} - 1\right) + 1$$

$$\frac{\partial \lambda_5}{dt} = \frac{\partial H}{\partial q_f}$$
$$= \lambda_5 \mu \left(\frac{Q_0}{q_n} - 1\right) + 1$$

6. Numerical Simulation

In these section described the numerical solutions to solve optimal control of the optimally system. The numerical simulations used to the optimal control u_1 and u_1, and the value from some parameters to shown the good result after or before given controlled system. The system influence by weight factor c_1 and c_2. The weight factor value of $c_1 = 0.5$, these can influence the acceleration of dilution rate of nutrient. The weight factor value of $c_2 = 0.5$, these can influence carbon dioxide accommodation to mobilization. Another parameters in the system are given in the **Table 2**. The parameters is getting from (Mairet etc, 2011) research. The initial conditions $s_0 = 3$, $q_{n0} = 0.5$, $x_0 = 0.02$, $q_{l0} = 0.25$ and , $q_{f0} = 0.5$. We getting the δ value is 5.08 from other research with combination natural light. Besides that, δ value still used for trial error in numerical simulations, but according to the (Sadya, Slamet, and Joni, 2012) research these value is best result

Table 2: *Parameters and Value* $(Mairet, etc. 2011)$

Parameters	Value
D	0.35
$\bar{\mu}$	1
ρ	0.095
β	4.8
γ	3
α	2.6
δ	5.08
Q_0	0.05
K_s	0.018

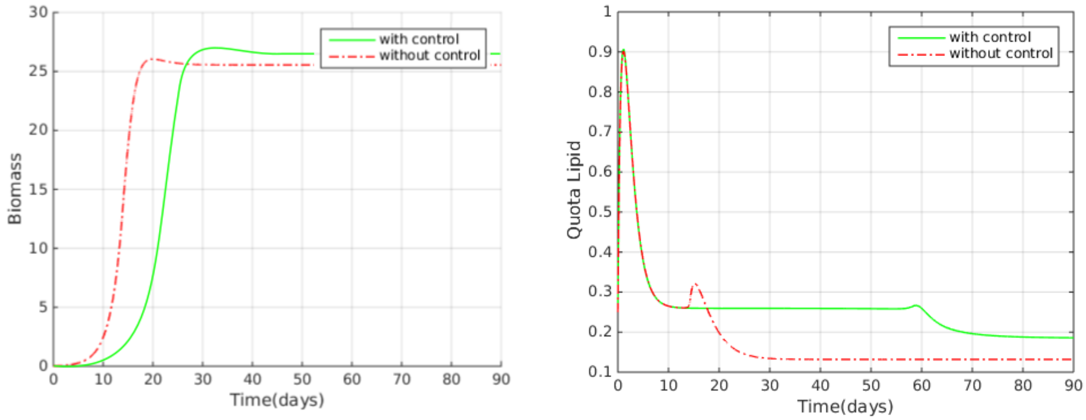

Figure 1: *The Result of Biomass and Quota Lipid before and aftre Controlled*

The numerical simulations used two conditions are without control applied and with control applied described at **Figure 1**. We given the same time conditions from $t_0 = 0$ to $t_1 = 90$ used time unit. The result of biomass without control conditions is 25,0146 $(gram/m2)$. After used the control, biomass increased into 26,1672 $(gram/m2)$. Therefore, the concentrations of biomass increased as 4,5678%. In another result of numerical simulation is lipid quota. The result of lipid quota without control conditions is 0.1283 $(gram/m2)$. After used the control, lipid quota increased into 0.1860 $(gram/m2)$. Therefore, the concentrations of lipid quota increased as $44,9727\%$.

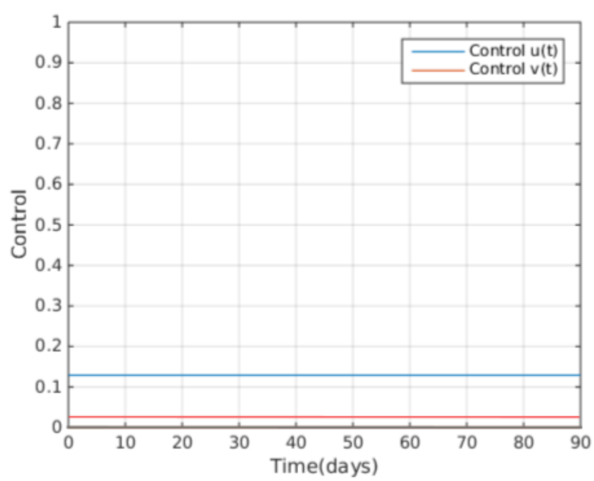

Figure 2: *Value of Control u_1 and u_2*

Figure 2 described about after simulation with objective function used u and v, thus obtain the value are $0,1292$ and $0,0121$. It's means the control has stable condition from start until finish.

7. Conclusion

- Using the Hamiltonian function the result derived of optimal control such as
$$u_1(t)^* = min\left(1, max\left(0, -\frac{\lambda_1(s - s_{in}) + \lambda_3 x}{c_1}\right)\right) \text{ and } u_2(t)^* = min\left(1, max\left(0, \frac{\frac{\lambda_3 \rho_m s x}{K_s + s} - \frac{\lambda_5 \rho_m s}{K_s + s}}{c_2}\right)\right)$$
- From the numerical simulation biomass increased $4,5678\%$ and quota lipid $44,9727\%$ after controlled the system.
- In the next research will be used another method to solve optimal control.

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