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MOVING AVERAGE CONTROL CHART FOR MONITORING PROCESS MEAN IN INAR(1) PROCESS WITH ZERO INFLATED POISSON

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Abstract

In this paper, the explicit formulas are proposed to evaluate the Average Run Length (ARL) of the Moving Average control chart (MA) for the first order integer-valued autoregressive with Zero Inflated Poisson mode (ZIPINAR(1)). The performance of MA and Exponentially Weighted Moving Average (EWMA) charts are compared. The results shown that, for $c_0 = 2$, the performance of MA chart is superior to EWMA chart. Especially, for upward shifts the performance of the MA chart gets better when the value of the span (w) decreases. However, for $c_0 = 5$, EWMA performs better than MA chart for all magnitudes of changes.

Keywords

Zero Inflated Poisson with first order integer-valued autoregressive model, Average Run Length, Moving Average control chart, *Exponentially* Weighted Moving Average (EWMA)

1. Introduction

Statistical Process Control Charts (SPC) are charts for manufacturing process control in order to make consistent products within standards and adapt the manufacturing process to have less variation. There are two types of control charts. The first is a variable control chart that controls manufacturing process with measurable outcome such as diameter of strand, and lifetime of light bulbs. There are several important variable control charts, such as $\overline{X}$, R, and S control charts with a main objective to control trends to the center of quality features. $\overline{X}$ control chart is used for controlling the mean of quality features. R and S control charts are used for controlling the distributions of quality features. The other type of control charts is an attribute control chart which is controlling the attribute quality feature of products that is not related to the standards of products and is not measurable such as blemish in clothes, broken light bulbs. Attribute control charts, such as broken ratio control chart (p Chart) and defective per unit control chart (c Chart), are popular to use for measuring quality. Then, a control chart that uses past history as factors is Cumulative Sum Control Chart (CUSUM Chart) (Page, 1954). Robert, 1959 proposed Exponentially Weighted Moving Average Control Chart (EWMA Chart) for detecting small change ($\delta \le 1.5\sigma$) (Montegomery, 2009). In 2004, Khoo proposed a Moving Average Control Chart (MA Chart) which calculated moving average from period of the value of the span (w). It could detect small changes and can be used with both continuous and discrete distributions. Generally, SPC is applied for manufacturing, computer science, telecommunications, financial and economics, epidemiology, and environmental areas. Therefore, the objective for the control charts is the ability to quickly detect changes in parameters of processes with minimum false alarm rate when a process is in-control and maximum true alarm rate when a process is out-of-control.

Countable process data mostly was found in the manufacturing process and the industrial services because it is easily to happen when considering the occurrence in each area. Countable data is useful for detecting changes in a process ranging from in-control process to out-of-control process. When a process changes, the efficiency of the process is loss and therefore an efficient tool is needed to detect the changes in order to make the process been back to normal. That tool is a control chart because it can detect changes in countable data especially the data with Poisson distribution. Therefore, a control chart that is suitable for this type of data is a c-control chart. Sometimes, this control chart is applied to the relational data with non-Poisson distribution. C-control chart is interested in current data, not the past data. Therefore, this chart is able to detect large changes but is not able to detect small changes (Montgomery, 2009). EWMA chart is suitable for the data with Poisson count process (Borror, Montgomery & Runger, 2006) and CUSUM chart is also good for the data with Poisson count process (Weiβ & Testik, 2009). Both charts are good for detecting small changes. Most researchers studied independent countable data but sometimes data can be autocorrelated. Therefore, a proposed model for autocorrelated data was Poisson integer valued with first order autoregressive model (PINAR(1)), which was similar to Autoregressive model with order 1 (AR(1)). AR(1) was a model for real number processing but PINAR(1) used binomial thinning operator to transform real number into integer number or countable data.

Al.Osh & Alzaid, 1987 developed PINAR(1) models by using binomial thinning operator. Applications for this model were used for infection rate of patients, flow rate of waterfall, number of insurance claims from insurance companies, and applications to queueing process (Cardinal, Roy & Lambert, 1999, Bockenholt, 2003).

Presently, automatic machines are widely used in manufacturing processes such that defective products are hardly happened. This causes the collected data for number of defectives to be mostly zero and this is called zero inflated data. The PINAR (1) model is not suitable for this type of data. Therefore, Jazi, Jones & Lai, 2012 introduced a model called Zero Inflated Poisson with first order integer-valued autoregressive model (ZIPINAR(1)). Later, Rakitzis, Weiβ & Castagliol, 2016 studied First-order integer valued of AR processes with zero inflated poisson, which was used with CUSUM charts. This model can track average changes in manufacturing processes and the authors discussed the influence of zero inflated data to the control chart.

The performance metric for control charts is Average Run Length (ARL), which consists to 2 types, in-control process or ARL_0 and out-of-control process or ARL_1. In a normal case, when a process is in control, ARL_0 should have high value. However, when a process is out of control, ARL_1 value should be minimum. There are several methods to find the ARL that are widely used and give accurate results as follows.

1) Monte Carlo Simulation (MC) is the method to calculate ARL under a specific event. This method is easy to calculate and the results are used as accuracy checks with ARL from other methods. However, the limitation for this method is long processing time.

2) Numerical Integration Equation (NIE) has limitation in finding the results from integral equations and can be only used with continuous distribution process.

3) Markov Chain Approach (MCA) uses a transition probability matrix. The results from this method are approximate values but this method uses less time than MC. This method can be used for processes with both continuous and discrete distributions.

From the above methods, no research proposed an explicit formula for INAR(1) model under zero inflated Poisson distribution. Therefore in this work, the explicit formula is proposed for Moving Average Control Chart (MA) to detect changes in means process for ZIPINAR(1) model and compares the efficiency of change detection between MA and EWMA charts.

2. The First Order Integer Valued Autoregressive with Zero Inflated Poisson Model

The first order integer-valued autoregressive (INAR(1)) model is perfectly suited for modeling count data. The INAR(1) model makes use of thinning operators for autocorrelated process of count data. This thinning operator is generated by counting series of Bernoulli distributed random variables. This model has many modifications and generalizations with respect to their order and marginal distribution, and it is quite suitable for use in counting certain random events. The INAR(1) model is defined by

$$N_t = \alpha \circ N_{t-1} + \varepsilon_t, \qquad (2)$$

where N_t is the observable count at time t, α is the first order integer-valued autoregressive parameter, $(\circ)$ is the thinning operation at time t performed independently of each other and ε_t is an innovation.

The ZIPINAR(1) model is the best fitting model for Poisson marginal distributions and ε_t follows the Zero Inflated Poisson distribution with mean $\dfrac{c(1-\phi)}{1-\alpha}$ then $\varepsilon_t \sim Poi(c,\phi)$ distribution with the zero inflated parameter (ϕ) where $\phi \in [0,1)$. According to the above situation, it can be modeled as the INAR(1) model, in which the expectation and variance of the ZIPINAR(1) model are

$$E[N_t]=\frac{c(1-\phi)}{1-\alpha} \text{ and } \mathrm{V}[N_t]=\frac{c(1-\phi)(1+\alpha+\phi c)}{(1-\alpha^2)}$$

Generally, the ZIPINAR(1) model could be changed in any unexpected occurrences, and then the change-point model of this process can be described by the following. Assume c_0 and $\frac{c_0(1-\phi)}{1-\alpha}$ are in control parameters, c_1 and $\frac{c_1(1-\phi)}{1-\alpha}$ are out of control parameters where δ is the magnitude of the shifts for out of control processes.

3. The Moving Average Control Chart for ZIPINAR(1) Model

A moving average control chart is based on unweighted moving average. Khoo, 2004 proposed the MA chart for number of nonconformities in an inspection unit of product. Suppose individual observations, $N_1, N_2, \ldots,$ are collected from ZIPINAR(1) model and be a sequence of independent identically distribution. The value of the span w at time i is defined as (Mongomery, 2009)

$$MA_i=\begin{cases}\frac{1}{i}\sum_{j=1}^{i}N_j; & i<w\\ \frac{1}{w}\sum_{j=i-w+1}^{i}N_j; & i\geq w.\end{cases} \tag{3}$$

When the process is in-control, the mean and variance of moving average are

$$E\left[MA_i\right]=\frac{c(1-\phi)}{1-\alpha} \tag{4}$$

and

$$Var\left(MA_i\right)=\begin{cases}\frac{c(1-\phi)(1+\alpha+\phi c)}{i\left(1-\alpha^2\right)} & ,i\leq w\\ \frac{c(1-\phi)(1+\alpha+\phi c)}{w\left(1-\alpha^2\right)} & ,i>w\end{cases} \tag{5}$$

The control limit of the MA chart are given

$$UCL/LCL=\begin{cases}\dfrac{c(1-\phi)}{1-\alpha}\pm k\cdot\sqrt{\dfrac{1}{i}\cdot\dfrac{c(1-\phi)(1+\alpha+\phi c)}{\left(1-\alpha^{2}\right)}} & ,i\le w\\ \dfrac{c(1-\phi)}{1-\alpha}\pm k\cdot\sqrt{\dfrac{1}{w}\cdot\dfrac{c(1-\phi)(1+\alpha+\phi c)}{\left(1-\alpha^{2}\right)}} & ,i>w\end{cases}\qquad(6)$$

where k is a coefficient of control limit and is determined based on a desired in-control ARL_0. The ZIPINAR(1) process of MA chart will signal to out-of-control when $MA_i < LCL$ or $MA_i > UCL.$

4. Explicit formula for ARL of MA chart for ZIPINAR(1) model

In this section, the explicit formula of MA chart for ZIPINAR(1) model is proposed. The explicit formula for evaluate ARL_0 and ARL_1 can be analytically derived by central limit theorem. Given o.o.c. is out-of-control limit (Khoo, 2004, Chananet, Areepong & Sukparungsee, 2015).

Let $ARL = n$ then

$$\frac{1}{ARL}=\frac{1}{n}P(o.o.c.\text{ signal at time } i<w)+\left[\frac{n-(w-1)}{n}\right]P(o.o.c.\text{ signal at time } i\ge w).\qquad(7)$$

According to the Eq. (7), the MA statistic for the case of signal to out-of control state is replaced by

$$=\frac{1}{n}\left\{\sum_{i=1}^{w}\left[P\left(\frac{\sum_{j=1}^{i}N_j}{i}>UCL_i\right)+P\left(\frac{\sum_{j=1}^{i}N_j}{i}<LCL_i\right)\right]\right\}$$

$$+\left[\frac{n-(2w-1)}{n}\right]\left\{P\left(\frac{\sum_{j=i-w+1}^{i}N_j}{w}>UCL_w\right)+P\left(\frac{\sum_{j=i-w+1}^{i}N_j}{w}<LCL_w\right)\right\}.\qquad(8)$$

Then, substitute the control limit of MA statistics from Eq. (6) into Eq. (8), which can be rewritten as

$$=\frac{1}{n}\left\{\sum_{i=1}^{w}\left[P\left(\frac{\sum_{j=1}^{i}N_j}{i}>\frac{c(1-\phi)}{1-\alpha}+k\cdot\sqrt{\frac{1}{i}\cdot\frac{c(1-\phi)(1+\alpha+\phi c)}{\left(1-\alpha^{2}\right)}}\right)\right.\right.$$
$$\left.\left.+P\left(\frac{\sum_{j=1}^{i}N_j}{i}<\frac{c(1-\phi)}{1-\alpha}-k\cdot\sqrt{\frac{1}{i}\cdot\frac{c(1-\phi)(1+\alpha+\phi c)}{\left(1-\alpha^{2}\right)}}\right)\right]\right\}$$

$$+\left[\frac{n-(w-1)}{n}\right]\left\{P\left(\frac{\sum_{j=i-w+1}^{i} N_j}{w} > \frac{c(1-\phi)}{1-\alpha} + k\cdot\sqrt{\frac{1}{w}\cdot\frac{c(1-\phi)(1+\alpha+\phi c)}{\left(1-\alpha^2\right)}}\right)\right.$$
$$\left.+P\left(\frac{\sum_{j=i-w+1}^{i} N_j}{w} < \frac{c(1-\phi)}{1-\alpha} - k\cdot\sqrt{\frac{1}{w}\cdot\frac{c(1-\phi)(1+\alpha+\phi c)}{\left(1-\alpha^2\right)}}\right)\right\}. \qquad (9)$$

The central limit theorem is used to derive the explicit formulas. Therefore, Eq. (9) can be rewritten as

$$\frac{1}{ARL} = \frac{1}{n}\left\{\sum_{i=1}^{w}\left[P\left(Z_1 > \frac{UCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{i\left(1-\alpha^2\right)}}}\right) + P\left(Z_1 < \frac{LCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{i\left(1-\alpha^2\right)}}}\right)\right]\right\}$$

$$+\left[\frac{n-(w-1)}{n}\right]\left\{P\left(Z_2 > \frac{UCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{w\left(1-\alpha^2\right)}}}\right) + P\left(Z_2 < \frac{LCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{w\left(1-\alpha^2\right)}}}\right)\right\}. \qquad (10)$$

According to Eq. (10), let

$$S = \sum_{i=1}^{w}\left[P\left(Z_1 > \frac{UCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{i\left(1-\alpha^2\right)}}}\right) + P\left(Z_1 < \frac{LCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{i\left(1-\alpha^2\right)}}}\right)\right]$$

and $$T = P\left(Z_2 > \frac{UCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{w\left(1-\alpha^2\right)}}}\right) + P\left(Z_2 < \frac{LCL_{i<w} - \frac{c(1-\phi)}{1-\alpha}}{\sqrt{\frac{c(1-\phi)(1+\alpha+\phi c)}{w\left(1-\alpha^2\right)}}}\right).$$

Then, the explicit formulas of ARL for MA chart is rewritten by substituting S and T into Eq. (10) as follows

$$\frac{1}{n} = \frac{1}{n}S + \frac{n-(w-1)}{n}T$$

$$n = \frac{1-S}{T} + (w+1).$$

As given that $ARL = n,$ then

$$ARL = \frac{(1-S)}{T} + (w-1), \quad w \neq \frac{n}{2} - 1. \qquad (11)$$

When the process is in-control the $ARL = ARL_0$, substitute the value of the parameter c with c_0. The proposed explicit formulas of ARL_0 of MA chart for ZIPINAR(1) can be rewritten as

$$ARL_0 = \left[1 - \sum_{i=1}^{w-1} \left[P\left(Z_1 > \frac{UCL_{i<w} - \frac{c_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_0(1-\phi)(1+\alpha+\phi c_0)}{i\left(1-\alpha^2\right)}}} \right) + P\left(Z_1 < \frac{LCL_{i<w} - \frac{c_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_0(1-\phi)(1+\alpha+\phi c_0)}{i\left(1-\alpha^2\right)}}} \right) \right] \right]$$

$$\times \left[P\left(Z_2 > \frac{UCL_{i<w} - \frac{c_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_0(1-\phi)(1+\alpha+\phi c_0)}{w\left(1-\alpha^2\right)}}} \right) + P\left(Z_1 < \frac{LCL_{i<w} - \frac{c_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_0(1-\phi)(1+\alpha+\phi c_0)}{w\left(1-\alpha^2\right)}}} \right) \right]^{-1} + (w-1). \qquad (12)$$

When the process is out-of-control then $ARL = ARL_1$, substitute the value of the parameter c with c_1. Explicit formula of the ARL_1 of the MA chart is

$$ARL_1 = \left[1 - \sum_{i=1}^{w-1} \left[P\left(Z_1 > \frac{UCL_{i<w} - \frac{c_1(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_1(1-\phi)(1+\alpha+\phi c_1)}{i\left(1-\alpha^2\right)}}} \right) + P\left(Z_1 < \frac{LCL_{i<w} - \frac{c_1(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_1(1-\phi)(1+\alpha+\phi c_1)}{i\left(1-\alpha^2\right)}}} \right) \right] \right]$$

$$\times \left[P\left(Z_2 > \frac{UCL_{i<w} - \frac{c_1(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_1(1-\phi)(1+\alpha+\phi c_1)}{w\left(1-\alpha^2\right)}}} \right) + P\left(Z_1 < \frac{LCL_{i<w} - \frac{c_1(1-\phi)}{1-\alpha}}{\sqrt{\frac{c_1(1-\phi)(1+\alpha+\phi c_1)}{w\left(1-\alpha^2\right)}}} \right) \right]^{-1} + (w-1), \qquad (13)$$

where c_1 is the out-of-control parameter. Note that if c_0 is changed, then $c = c_1$ where $c_1 = (1+\delta)c_0$ and the shift parameter (δ) as varied from 0.1, 0.3, 0.5, …,1.0, 1.5, 2.0.

5. The Numerical Results

In this section, the explicit formulas for ARL_0 and ARL_1 of MA chart for ZIPINAR (1) process are proposed. The numerical results for ARL_0 and ARL_1 of MA chart are calculated from Eq. (12) and (13). In case of in-control parameter of MA chart, the value of the span (w) were 2, 5, 10, 15 and 20. when in-control parameter value (ARL_0) was 370, the coefficient control limit (k) equal 3. In case of in-control parameter of ZIPINAR(1) process, c_0 equal 2, 5 and α equal 0.1. In case of out-of-control parameter of ZIPINAR(1) process, c_1 equal $(1+\delta)c_0$, and the parameter magnitude values of δ are 0.1, 0.3, 0.5,…, 1.0, 1.5, 2.0.

From Table 1 and 2, when c_0 = 2, MA chart shows better performance than EWMA chart in every level changed. For MA chart, the observation is that when the change level is increased, the optimal of w value is decreased. In contrast from Table 3 and 4, when c_0=5, EWMA chart performs better than MA chart for all levels of changes. These results show that the performance of charts depends on the data characteristic or in control parameters, i.e. c_0, ϕ, α .

Table 1: *ARL comparison of the MA chart using explicit formulas with the EWMA chart for ZIPINAR(1) model given c=2* $\phi = 0.5$ $\alpha = 0.1$

Shift	MA					EWMA
	w=2	*w=5*	*w=10*	*w=15*	*w=20*	L=4.23125
0	370.398	370.398	370.398	370.398	370.398	370.794 ± 1.498*
0.1	289.492	284.634	276.971	269.814	**263.128**	343.297 ± 1.393
0.3	179.123	160.409	136.741	119.543	**106.737**	149.785 ± 0.711
0.5	115.075	92.498	70.040	57.296	**49.659**	65.488 ± 0.271
0.7	77.501	56.933	40.285	32.715	**29.185**	40.752 ± 0.144
0.9	54.709	37.631	26.054	21.907	**20.719**	30.133 ± 0.095
1.0	46.726	31.365	21.776	18.808	**18.366**	27.128 ± 0.081
1.5	24.367	15.464	**11.800**	11.872	13.158	17.979 ± 0.046
2.0	15.136	9.739	**8.537**	9.611	11.314	13.479 ± 0.032

*is standard deviation of ARL

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Table 2: *ARL comparison of the MA chart using explicit formulas with the EWMA chart for ZIPINAR(1) model given c=2* $\phi = 0.7$ $\alpha = 0.1$

Shift	MA					EWMA
	w=2	***w=5***	***w=10***	***w=15***	***w=20***	**L=3.3473**
0	370.398	370.398	370.398	370.398	370.398	370.049 ± 1.312*
0.1	291.152	288.670	284.674	280.849	277.189	$\mathbf{248.793 \pm 1.012}$
0.3	186.406	175.890	160.954	148.644	138.424	$\mathbf{103.106 \pm 0.495}$
0.5	125.354	111.061	93.958	81.579	72.997	$\mathbf{57.032 \pm 0.229}$
0.7	88.339	73.803	58.506	49.465	43.961	$\mathbf{38.044 \pm 0.144}$
0.9	64.931	51.677	39.446	33.251	30.118	$\mathbf{28.700 \pm 0.103}$
1.0	56.459	44.041	33.258	28.216	25.964	$\mathbf{25.525 \pm 0.090}$
1.5	31.516	23.071	17.532	**16.047**	16.262	16.277 ± 0.085
2.0	20.379	14.639	**11.838**	11.887	13.025	12.086 ± 0.040

*is standard deviation of ARL

Table 3: *ARL comparison of the MA chart using explicit formulas with the EWMA chart for ZIPINAR(1) model given c=5* $\phi = 0.5$ $\alpha = 0.1$

Shift	MA					EWMA
	w=2	***w=5***	***w=10***	***w=15***	***w=20***	**L=5.0643**
0	370.398	370.398	370.398	370.398	370.398	370.024 ± 1.522*
0.1	335.314	333.884	331.554	329.289	327.088	$\mathbf{306.627 \pm 1.533}$
0.3	273.680	264.613	250.863	238.630	227.717	$\mathbf{216.892 \pm 1.063}$
0.5	223.693	205.771	182.271	163.833	149.145	$\mathbf{82.425 \pm 0.333}$
0.7	182.320	159.202	131.448	112.320	98.672	$\mathbf{45.920 \pm 0.150}$
0.9	149.797	123.711	95.934	78.968	68.035	$\mathbf{32.911 \pm 0.094}$
1.0	136.134	109.439	82.589	67.096	57.580	$\mathbf{29.003 \pm 0.078}$
1.5	87.048	62.294	42.912	34.260	30.218	$\mathbf{18.683 \pm 0.044}$
2.0	58.794	38.789	26.022	21.659	20.477	$\mathbf{13.782 \pm 0.031}$

*is standard deviation of ARL

Table 4: *ARL comparison of the MA chart using explicit formulas with the EWMA chart for ZIPINAR(1) model given c=5* $\phi = 0.7$ $\alpha = 0.1$

Shift	MA					EWMA
	w=2	*w=5*	*w=10*	*w=15*	*w=20*	L=3.60779
0	370.398	370.398	370.398	370.398	370.398	370.281 ± 1.427*
0.1	335.826	335.153	334.049	332.968	331.911	**260.068 ± 1.139**
0.3	277.077	272.666	265.697	259.178	253.080	**94.069 ± 0.456**
0.5	229.985	221.113	207.890	196.357	186.264	**47.398 ± 0.208**
0.7	192.233	179.626	162.096	147.986	136.499	**30.629 ± 0.127**
0.9	161.897	146.701	127.062	112.486	101.458	**22.816 ± 0.090**
1.0	149.010	132.940	112.922	98.644	88.214	**20.225 ± 0.078**
1.5	101.362	84.030	65.879	55.124	48.496	**13.009 ± 0.048**
2.0	72.260	56.409	42.139	35.008	31.383	**9.482 ± 0.034**

*is standard deviation of ARL

6. Conclusion

The explicit formulas of the ARL_0 and ARL_1 were derived of the Moving Average chart for the ZIPINAR(1) process. The suggested formulas are easy to calculate and program. Obviously, the computational time for evaluating the suggested formulas is much less 1 second. Thus, it is suggested that the explicit formulas of the ARL of CUSUM chart can be applied to real data, empirical data, and real-world situations applications such as in economics, finance, environmental, etc.

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