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ROBUSTNESS OF GENERALLY WEIGHTED MOVING AVERAGE SIGNED – RANK CONTROL CHART FOR MONITORING A SHIFT OF SKEW PROCESSES

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Abstract

A distribution-free control charts play a virtual role in quality control chart because it is not necessary to know the assumption of specific distribution for any process. In this article, the robustness of a generally weighted moving average based on signed-rank (GWMA-SR) control charts are intensive studied for skew processes. In addition, the GWMA-SR control chart is compared the performance to detecting a shift in process mean based on GWMA with sign statistic (GWMA-SN) and exponentially weighted moving average with signed-rank statistic (EWMA-SR) control charts. The numerical results using Monte Carlo simulation found that the performance of GWMA-SR chart robust to skew process and perform as good as the benchmark charts. Furthermore, the GWMA-SR chart is superior to existing chart in many situations.

Keywords

Distribution-Free, Signed-Rank, Sign Statistic, Nonparametric Chart

1. Introduction

Statistical Process Control (SPC) charts are widely used to monitor, measure, control and improve quality of production in many areas of application, i.e., finance and economics, industry and manufacturing (Butlet and Stefani (1994)), epidemiology and health sciences, environmental statistics and other fields. Nowadays, they are emerged in order to design an efficient control charts for quick detecting a small shift in the interested characteristics of process such as location or shape parameter. Typically, the traditional Shewhart chart is popularly known and most widely used in industry but its limitation to detect large shifts. Therefore, the time-varying charts are effective alternatives to the traditional chart for detecting a small shift such as Exponentially Weighted Moving Average (EWMA) was first presented by Roberts, 1959 and Generally Weighted Moving Average (GWMA) by Sheu and Lin, 2003 that those latter can overcome this limitation. However, they are more feasible and relax when processes are violated from normal distribution (Borror et al., 1999). Later, Sheu and Chiu (2007) presented the GWMA control chart, which were found to be performing well when a level of change in a mean of the production process is large and Yada et al. (2015) has estimated the average run length Markov chain method to detect changes in the number of nonconformities.

Most of the data collected from the process is normal distributed but in practice, the data from the process may have non-normal distribution (Alkahtani (1991), Ergashev (1993), Borror (1999), Areepong and Novikov (2008)), or unknown parameter and distribution free. Later, there are many literatures to propose the nonparametric control charts as an effective alternative to parametric control chart (Sukparungsee (2012), Khaliq et al. (2016)). In 1991, Amin and Searcy presented a EWMA chart using the signed-rank statistics, known as the EWMA-SR chart for detecting the median of the process. Graham et al. (2011a) studied the efficiency and applicability of real data to EWMA-SR charts. In the same year, Graham et al. (2011b) proposed the EWMA control chart under the sign statistic called EWMA-sign. Lu (2015) has proposed a GWMA chart under the sign statistic called GWMA-SN which outperforms EWMA-SN for small changes and neither of these charts performs much differently for large changes. Furthermore, non-parametric GWMA control charts outperform the parametric GWMA control chart when the process is normal and not normal. In 2010, Yang et al. proposed a non-parametric cumulative control chart (Non-CUSUM) which does not require the process parameters. The result found that Non-CUSUM charts are more effective in detecting better than the parametric CUSUM control chart. Yang et al. (2001) presented non-parametric exponentially moving average (EWMA Sign) and Arcsine EWMA Sign (EWMA Arcsine) control charts for monitoring a change in process which the performance of nonparametric EWMA is superior to parametric EWMA control chart. Recently, Chakraborty, Chakraborti, Human, & Balakrishnan (2016) proposed a Generally Weighted Moving Average-Wilcoxon Signed rank to detect symmetry process.

In this paper, the robustness of the GWMA Signed-rank (GWMA-SR) control chart are investigated to the asymmetry process and compare the performance with EWMA Signed-rank (EWMA - SR) and GWMA Sign (GWMA - SN) control charts.

2. Control charts and theirs properties

This research relies on the following related theories are statistical parameters and nonparametric control charts. They consist of three control charts are GWMA, GWMA Signed-rank (GWMA-SR) and GWMA Sign (GWMA-SN) control charts. The details are as follows:

2.1 Generally Weighted Moving Average control chart (GWMA chart)

The GWMA chart was initially presented by Shey and Lin (2003) and intensive studied by Chiu (2009) is weighted moving average of sequential historical observations. Since, each observation has differently weighted that can be decreased from the present time period to the past time periods. Consequently, it could be reflected to the important observations on recent process. The GWMA chart was extended and developed from EWMA chart by modifying an adjustment smoothing constant (α). The GWMA chart coincides the EWMA chart when the weighted historical observation constant is given as $q = 1-\lambda$ and $\alpha = 1$. Let $X_1, X_2, ..., X_t, ...$ be observed independent random variables. The GWMA statistic is

$$G_t = \sum_{i=1}^{t} (q^{(i-1)^\alpha} - q^{i^\alpha}) X_{t-i+1} + q^{t^\alpha} G_0. \tag{1}$$

Eq.(1) can be rewritten to Eq.(2) by taking geometric series as

$$G_t = \frac{(1-q)(q-1)-(q-1)q(q-1)}{(q-1)(1-q)} X_{t-i+1} + q^\alpha G_0 \tag{2}$$

where G_t is the GWMA statistic at time t^{th}, where the initial statistic value $G_0 = \mu_0$

X_{t-i+1} is the observations of skew process at the $t-i+1^{th}$, $t = 2,3,...$

q is a weighted historical observations constant $(0 < q \le 1)$

α is an adjustment smoothing constant $(\alpha > 0)$.

Mean and variance of GWMA statistic are $E(G_t) = \mu_0$ and $Var(\mathrm{G}_t) = \sigma^2_{G_t} = Q_t \sigma^2_x$, respectively.

Therefore, the control limits of GWMA chart are

$$UCL/LCL = \alpha_0 \pm L\sigma_X \sqrt{Q_t}$$

where $Q_t = \sum_{i=1}^{t} (q^{(i-1)^\alpha} - q^{i^\alpha})^2$ and L is the coefficient of the width GWMA control limits.

2.2 Generally Weighted Moving Average Signed-rank control chart (GWMA-SR chart)

This control chart was first proposed by Chakraborty et al. (2016) which combined parametric GWMA control chart with nonparametric statistic as signed-rank so called *Generally Weighted Moving Average Signed-rank control chart (GWMA-SR chart).* It was developed from GWMA control chart and the performance of the GWMA is superior to EWMA chart.

Assume that a quality characteristic of interest, X have continuous and asymmetry around θ. In this research, the asymmetry processes are studied then given θ to be median and its known value because it is a more robust measure and a better representative of a central value of a distribution than the mean. Suppose X_{ij}, denote the j^{th} observation in the i^{th} random sample (rational subgroup) of sample size $n > 1$ and R^+_{ij} denote the ranks of the absolute difference $|X_{ij} - \theta_{ij}|$, for $j = 1, 2, ..., n$, within the i^{th} subgroup. The signed-rank (SR) statistic can be defined as

$$\mathrm{SR_i} = \sum_{\mathrm{j=1}}^{n} sign\left(\mathrm{x_{ij}} - \theta_0\right) R^+_{ij},\ i = 1, 2, 3, ... \tag{3}$$

where *sign(x)* = +1 or -1 if *x>0* or *x<0*, respectively. Then, it is verified that $SR_i = T^+_i - T^-_i$ is the difference between the sum of the ranks of the $|X_{ij} - \theta_{ij}|$, corresponding to positive and negative differences, respectively, within the i^{th} subgroup. In addition, SR_i can be written as $SR_i = 2T^+_i - n(n+1)/2$ because of the sum of all ranks within a sample $T^+_i - T^-_i = n(n+1)/2$.

The GWMA-SR statistics can be simplified as

$$Z_t = \sum_{i=1}^{t} \left(q^{(i-1)^\alpha} - q^{i^\alpha}\right) SR_{t-i+1};\ t = 1, 2, 3, \ldots \tag{4}$$

where $0 \le q < 1$ and $\alpha > 0$. Note that the GWMA-SR chart coincides to the EWMA-SR chart when $\alpha = 1$ and $q = 1$, where $0 < \lambda \le 1$ is the smoothing parameter of the EWMA chart. The EWMA-SR chart further reduces to the Shewhart-SR chart when $q = 0$ and $\alpha = 1$. The GWMA chart can therefore be presented as a generalisation of both EWMA and Shewhart charts with an additional parameter α that provides more flexibility in control chart designing.

The in-control expected value and variance of the statistic Z_t are given by

$$E\left(Z_t \mid in\ control\right) = \sum_{i=1}^{t}\left(q^{(i-1)^\alpha} - q^{i^\alpha}\right)E\left(SR_{(t-i+1)}\right) = 0 \tag{5}$$

and

$$Var\left(Z_t \mid in\ control\right) = \sum_{i=1}^{t}\left(q^{(i-1)^\alpha} - q^{i^\alpha}\right)^2 \operatorname{var}\left(SR_{(t-i+1)}\right) = \frac{n(n+1)(2n+1)}{6}Q_t, \tag{6}$$

where $Q_t = \sum_{i=1}^{t}\left(q^{(i-1)^\alpha} - q^{i^\alpha}\right)^2$ is the sum squares of the weights. Equation (5) and (6) are the results of mean and variance of the well-known Wilcoxon SR test statistic coupled with the properties of the EWMA charting statistic. The exact time-varying (and symmetrically placed) upper control limit (UCL) and lower control limit (LCL) of the GWMA-SR chart are given by

$$UCL/LCL = \pm L\sqrt{\frac{n(n+1)(2n+1)}{6}Q_t} \tag{7}$$

where $L > 0$ is a coefficient of the width GWMA-SR control limits.

The Equation (6) can be rewritten as asymptotic variance by given $\lim_{t\to\infty}\operatorname{var}\left(Z_t\right) = \frac{n(n+1)(2n+1)}{6}Q$, where $Q_t = \lim_{t\to\infty} Q_t$, which is an increasing function of t and converges as $t \to \infty$. Then, the steady-state control limits and the center line are given by

$$UCL/LCL = \pm L\sqrt{\frac{n(n+1)(2n+1)}{6}Q}$$

2.3 Generally Weighted Moving Average Sign control chart (GWMA-SN chart)

Assume that a quality characteristic, X, has a target value T. Let $Y = X - T$, be the different value between observation and target value then it could be positive or negative value as statistic sign test in nonparametric. The process proportion $p = P(Y > 0)$, where $p = 0.5$ for the in-control process. If the process is out-of-control (the observations deviate from the target value), then process proportion has changed to $p \neq 0.5$. In order to detect, the deviation from the process target at any unexpected times, a random sample of size. The statistics Y_j is defined as follows:

$$Y_j = X_j - T \quad \text{and} \quad I_j = \begin{cases} 1 & ; Y_j > 0 \\ 0 & ; Other \end{cases} \tag{8}$$

Let M be the sum number of $Y_j > 0$, then $M = \sum_{j=0}^{n} I_j$ will follow a binomial distribution with parameter $(n, 0.5)$ when the process is in-control. Therefore, the nonparametric GWMA statistic at time *t* can be defined as

$$G_t = \sum_{i=1}^{t} \left(q^{(i-1)^\alpha} - q^{i^\alpha} \right) M_{t-i+1} + q^{i^\alpha} T; \quad 0 \leq \text{q} < 1, \quad 0 < \alpha \leq 1 \tag{9}$$

where M is the i^{th} sequentially recorded number of $Y_j > 0$, from the process. Let the initial value of the statistic of GWMA sign control chart $G_0 = T = n/2$.

The expected value and variance of G_t are following:

$$E(G_t \mid in\ control) = E\left(\sum_{i=1}^{t} \left(q^{(i-1)^\alpha} - q^{i^\alpha} \right) M_{t-i+1} + q^{i^\alpha} T \right) = \frac{n}{2} \tag{10}$$

and

$$\begin{aligned} Var(G_t \mid in\ control) &= Var\left(\sum_{i=1}^{t} \left(q^{(i-1)^\alpha} - q^{i^\alpha} \right) M_{t-i+1} + q^{i^\alpha} T \right) \\ &= \sum_{i=1}^{t} \left(q^{(i-1)^\alpha} - q^{i^\alpha} \right)^2 \times Var(M_i) \\ &= Q.\frac{n}{4} \end{aligned} \tag{11}$$

where $Q_t = \sum_{i=1}^{t}\left(q^{(i-1)^{\alpha}} - q^{i^{\alpha}}\right)^2$ is an asymptotic value when $t \to \infty$. Therefore, the control limits for the GWMA sign control chart are as follows:

$$\begin{aligned} UCL_{GWMA-SN} &= \frac{n}{2} + L\sqrt{\left(\frac{n}{4}\right)Q} \\ LCL_{GWMA-SN} &= \frac{n}{2} - L\sqrt{\left(\frac{n}{4}\right)Q} \end{aligned} \tag{12}$$

where L is the width of control limit of GWMA-SN chart.

2.4 The Average Run Length

Let $E_{\theta}(\cdot)$ denote the expectation under distribution $F(x, p_0)$ that the change-point occurs at point θ. In the literature on quality control the quantity $E_{\infty}(\tau_b) = A$ is the so-called "Average Run Length," which is a method frequently used in SPC charts for evaluation of the performance of various control charts. There are various characteristics that correspond with the performance of SPC chart, however, the average run length is still the most popular and commonly used characteristic for evaluating the performance of the SPC chart. There are two cases of the average run length as follows:

The first one shows that the performance of the SPC chart is in-control. It is called "in-control Average Run Length" or "ARL_0," denoted as

$$ARL_0 \equiv E_{\upsilon}(\tau) = T, \quad \upsilon = \infty \tag{13}$$

where υ is a change-point time, τ is the first exit time, $E_{\upsilon}(\tau)$ is the expectation under distribution $F(x, p_0)$ that the change-point occurs at point υ, and T is constant.

The next one shows that the performance of the SPC chart is out-of-control. It is called "out-of-control Average Run Length" or "ARL_1," which depends on parameter υ. The ARL_1 was denoted as

$$ARL_1 \equiv E_{\upsilon}(\tau - \upsilon + 1 | \tau \geq \upsilon), \ \upsilon = 1, \tag{14}$$

where $E_{\upsilon}(\tau - \upsilon + 1 | \tau \geq \upsilon)$ is the expectation under distribution $F(x, p \neq p_0)$ that the change-point occurs at point υ, respectively.

The expression of the ARL_0 and ARL_1 for the control chart in detecting the value of the mean shift in an identically and independent distribution of observation processes has been studied in many literatures. They are two conflicting criteria that had to be balanced in the control charts that were used as the basis for the comparison of different charts throughout this research.

3. The Numerical Results and Performance Comparison

The performance of nonparametric GWMA-SR, EWMA-SR and GWMA-SN control charts are compared when observation are from exponential, lognormal gamma and Weibull distributions. The numerical results of approximations are based on Monte Carlo simulation which repeated 50,000 times. The value of ARL_0 is given to 370, sample sizes $n=5$ and the magnitudes of change are $\delta=(0.05,\ 0.1,\ 0.25,\ 0.5,\ 0.75,\ 1.00)$. The weighted values of GWMA chart $q=0.8$ and $\alpha=0.5$ and the weighted parameter of EWMA statistic $\lambda=0.2$ and $\alpha=1$. The best performance of any control chart will give a minimum ARL_1 as shown on Table 1, 2, 3 and 4 for the case of exponential, lognormal gamma and Weibull distribution, respectively.

Table 1: *Average Run Lengths for GWMA-SR, GWMA-SN and EWMA-SR charts for exponential(1) process with q=0.8, n=5 and ARL_0=370*

δ	Chart	α	L	ARL_1
0.00	GWMA-SR	0.5	3.255	370.02
	EWMA-SR	1.0	3.164	370.23
	GWMA-SN	0.5	39.32	370.90
0.05	GWMA-SR	0.5	3.255	**197.189**
	EWMA-SR	1.0	3.164	221.164
	GWMA-SN	0.5	39.32	214.804
0.10	GWMA-SR	0.5	3.255	**117.071**
	EWMA-SR	1.0	3.164	129.398
	GWMA-SN	0.5	39.32	128.182
0.25	GWMA-SR	0.5	3.255	46.954
	EWMA-SR	1.0	3.164	**41.840**
	GWMA-SN	0.5	39.32	65.888
0.50	GWMA-SR	0.5	3.255	22.361

	EWMA-SR	1.0	3.164	**15.814**
	GWMA-SN	0.5	39.32	56.646
0.75	GWMA-SR	0.5	3.255	14.841
	EWMA-SR	1.0	3.164	**9.629**
	GWMA-SN	0.5	39.32	52.094
1.00	GWMA-SR	0.5	3.255	11.400
	EWMA-SR	1.0	3.164	**7.127**
	GWMA-SN	0.5	39.32	49.089

Bold number means the minimum ARL_1.

Table 2: *Average Run Lengths for GWMA-SR, GWMA-SN and EWMA-SR charts for lognormal(1, 1) process with q=0.8, n=5 and ARL_0=370.*

δ	Chart	α	L	ARL_1
0.00	GWMA-SR	0.5	3.34	371.02
	EWMA-SR	1.0	3.22	370.91
	GWMA-SN	0.5	181.63	369.67
0.05	GWMA-SR	0.5	3.34	**216.49**
	EWMA-SR	1.0	3.22	234.23
	GWMA-SN	0.5	181.63	233.28
0.10	GWMA-SR	0.5	3.34	**115.34**
	EWMA-SR	1.0	3.22	123.18
	GWMA-SN	0.5	181.63	142.75
0.25	GWMA-SR	0.5	3.34	37.78
	EWMA-SR	1.0	3.22	**29.84**
	GWMA-SN	0.5	181.63	70.47
0.50	GWMA-SR	0.5	3.34	14.60
	EWMA-SR	1.0	3.22	**9.09**
	GWMA-SN	0.5	181.63	63.63
0.75	GWMA-SR	0.5	3.34	8.55
	EWMA-SR	1.0	3.22	**4.99**
	GWMA-SN	0.5	181.63	59.70

1.00	GWMA-SR	0.5	3.34	6.17
	EWMA-SR	1.0	3.22	**3.49**
	GWMA-SN	0.5	181.63	56.78

Bold number means the minimum ARL_1.

Table 3: *Average Run Lengths for GWMA-SR, GWMA-SN and EWMA-SR charts for gamma(4, 1) process with q=0.8, n=5 and ARL_0=370.*

δ	Chart	α	L	ARL_1
0.00	GWMA-SR	0.5	2.859	370.83
	EWMA-SR	1.0	2.900	369.79
	GWMA-SN	0.5	163.89	370.73
0.05	GWMA-SR	0.5	2.859	**103.10**
	EWMA-SR	1.0	2.900	133.08
	GWMA-SN	0.5	163.89	134.95
0.10	GWMA-SR	0.5	2.859	**40.24**
	EWMA-SR	1.0	2.900	44.74
	GWMA-SN	0.5	163.89	74.44
0.25	GWMA-SR	0.5	2.859	10.49
	EWMA-SR	1.0	2.900	**8.36**
	GWMA-SN	0.5	163.89	63.27
0.50	GWMA-SR	0.5	2.859	4.10
	EWMA-SR	1.0	2.900	**3.06**
	GWMA-SN	0.5	163.89	57.22
0.75	GWMA-SR	0.5	2.859	2.73
	EWMA-SR	1.0	2.900	**2.20**
	GWMA-SN	0.5	163.89	54.20
1.00	GWMA-SR	0.5	2.859	2.26
	EWMA-SR	1.0	2.900	**2.03**
	GWMA-SN	0.5	163.89	52.65

Bold number means the minimum ARL_1.

Table 4: *Average Run Lengths for GWMA-SR, GWMA-SN and EWMA-SR charts for Weibull (0.5, 1) process with q=0.8, n=5 and ARL_0=370*

δ	Chart	α	L	ARL_1
0.00	GWMA-SR	0.5	3.76	371.20
	EWMA-SR	1.0	3.51	370.26
	GWMA-SN	0.5	20.94	370.23
0.05	GWMA-SR	0.5	3.76	**262.88**
	EWMA-SR	1.0	3.51	270.44
	GWMA-SN	0.5	20.94	286.45
0.10	GWMA-SR	0.5	3.76	**199.68**
	EWMA-SR	1.0	3.51	206.51
	GWMA-SN	0.5	20.94	218.94
0.25	GWMA-SR	0.5	3.76	111.77
	EWMA-SR	1.0	3.51	**105.78**
	GWMA-SN	0.5	20.94	108.80
0.50	GWMA-SR	0.5	3.76	63.74
	EWMA-SR	1.0	3.51	**49.66**
	GWMA-SN	0.5	20.94	69.17
0.75	GWMA-SR	0.5	3.76	44.96
	EWMA-SR	1.0	3.51	**30.76**
	GWMA-SN	0.5	20.94	62.05
1.00	GWMA-SR	0.5	3.76	35.47
	EWMA-SR	1.0	3.51	**22.26**
	GWMA-SN	0.5	20.94	58.33

Bold number means the minimum ARL_1.

4. Conclusion and Discussion

In this research, the nonparametric GWMA-SR, EWMA-SR and GWMA-SN control charts are studied the robustness of those control charts to skew processes and the performance in order to detect the deviation from the target value of process are compared. These nonparametric control charts are based on signed-rank and sign tests which binomial distribution will be concerned. The numerical results are carried on by the Monte Carlo simulation with $5*10^4$ times repeated. We found that the GWMA-SR chart is superior to EWMA-SR and GWMA-SN control charts for small magnitudes of change $(\delta < 0.25)$ for all skew distributions of process. Otherwise, the EWMA-SR chart performs better than other charts for moderate and large shifts $(\delta \geq 0.25)$.

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